

1967

Survey of needs and development of an instructional unit in mathematics for industrial arts students

Lawrence LeRoy Stokka
Iowa State University

Follow this and additional works at: <https://lib.dr.iastate.edu/rtd>



Part of the [Educational Assessment, Evaluation, and Research Commons](#), and the [Other Education Commons](#)

Recommended Citation

Stokka, Lawrence LeRoy, "Survey of needs and development of an instructional unit in mathematics for industrial arts students" (1967). *Retrospective Theses and Dissertations*. 16358.
<https://lib.dr.iastate.edu/rtd/16358>

This Thesis is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.

SURVEY OF NEEDS AND DEVELOPMENT OF AN
INSTRUCTIONAL UNIT IN MATHEMATICS
FOR INDUSTRIAL ARTS STUDENTS

by

Lawrence LeRoy Stokka

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
MASTER OF SCIENCE

Major Subject: Industrial Education

Signatures have been redacted for privacy

Iowa State University
Of Science and Technology
Ames, Iowa

1967

TABLE OF CONTENTS

	Page
INTRODUCTION	1
METHOD OF PROCEDURE	7
FINDINGS	15
DISCUSSION	23
INDUSTRIAL ARTS MATHEMATICS	28
SUMMARY	87
LITERATURE CITED	90
APPENDIX A INDUSTRIAL ARTS MATHEMATICS TEST	91
APPENDIX B QUESTIONNAIRE FOR INDUSTRIAL ARTS TEACHERS	93
APPENDIX C SUPERVISOR'S QUESTIONNAIRE	94

INTRODUCTION

The term "mathematics", as used in this study, is a broad term which includes any problem solving using the operations of addition, subtraction, multiplication, or division of numbers in their various forms. This is sometimes referred to as arithmetic. It also includes the use of equations, variables, and geometric relationships, sometimes referred to as mathematics.

Industrial arts, as used in this study refers to the general education, elective courses offered in some degree by most high schools. These courses do not have the objective of preparing one for a particular job. Rather, they are designed to be exploratory and to broaden one's knowledge of industrial products and processes. Also, industrial arts courses are designed to stimulate one's vocational interests.

High school industrial arts students are required to take mathematics throughout the elementary and junior high school years plus one additional year sometime during the period of grades nine through twelve. However, with these years of study of mathematics, many high school students of industrial arts do not have the ability to solve even the very elementary mathematical problems which confront them in the activities of industrial arts courses.

Several years ago a Des Moines industrial firm offered a course in machine shop to its semi-skilled employees, six hours of which were devoted to mathematics. The purpose of the course was to up-grade the employee and allow him to prepare himself for advancement to a higher paying position. The employees who took advantage of the course had several years of industrial experience and knew the value of mathematical ability in their work and the necessity of it for advancement.

The motivation to learn mathematics was very strong in this group.

Chrysler Corporation, in the introduction to their publication, *Math Problems From Industry* (1) makes the following statements:

A student writes to ask us, "Is math really necessary in business?"

A teacher writes, "Please send illustrations of how mathematics is used at Chrysler Corporation, so that I can give my students actual problems of the type which they might be required to solve after they leave school."

Multiply these requests many times and you have our reason for publishing "*Math Problems From Industry*" — to meet a definite need. We, at Chrysler Corporation, know that mathematics is one of the most widely used tools in industry. In our company, draftsmen, technicians, foremen, skilled craftsmen, engineers, and many others use it daily as they solve a great variety of technical problems.

The above statements were followed by fifty-one pages of practical mathematics problems, typical of problems which might confront one in industry. All of these problems were typical, also, of problems which might confront an industrial arts student.

Chrysler Corporation received so many letters from teachers seeking material on mathematics, that they went to the trouble and expense of publishing a booklet on mathematics and made it available, free of charge, in classroom sets. This fact indicates there is a lack of material for teaching mathematics related to industry.

The high degree of interest in industry related mathematics shown by the employees of the Des Moines firm previously mentioned, and by the many requests received by Chrysler Corporation indicates a great desire to learn mathematics if

the teaching can be directly related to the person's work or interests.

The purpose of this study was to determine the areas of weakness in mathematical ability among high school industrial arts students and semi-skilled workers in industry. On the basis of this determination, a text was developed for teaching related mathematics to industrial arts students.

The following assumptions were made in undertaking this study:

1. That many of the students of industrial arts courses will be employed in the semi-skilled positions in industry.
2. That those employed in the semi-skilled positions in industry, in general, are very weak in ability to handle simple mathematical problems related to their work.
3. That the lack of ability in mathematics is a result of low interest in required mathematics courses.
4. That the mathematical ability of these people could be greatly improved if they had a unit in mathematics in the industrial arts courses in high school which was highly related to the work done in industrial arts courses and in industry.
5. That there would be greater desire to learn mathematics when applied to an area of interest or need.
6. That a desire exists among industrial arts teachers to obtain a text suitable for teaching related mathematics in industrial arts classes.

The investigation of the mathematical ability of students was limited to high school industrial arts students. It was further limited by excluding students in

drafting courses. It was felt that drafting classes contain many students who are planning a career in engineering, drafting being a background course for engineering. These students would, therefore, be above average in mathematical ability. The inclusion of this group would undoubtedly bias the investigation of mathematical ability.

This study also excludes vocational classes. Vocational programs in the wood and metal areas usually contain a course in related mathematics which is very thorough. The mathematics text which will be developed on the basis of this study would not be suitable for vocational classes.

Examination of textbooks available for use in industrial arts and vocational courses will quickly reveal a severe lack of material suitable for teaching mathematics in industrial arts.

The book "Simplified Sheet Metal Mathematics" (4) is a good example of thoroughness of mathematics texts designed for specific vocational areas. This text includes instructions in the very basic fundamentals of arithmetic and then proceeds step-by-step to advance through the more complex problems of the sheet metal trade. It includes sections on algebra, geometry, trigonometry and special formulas required for sheet metal work.

Fierer's (2) book, is representative of textbooks for industrial arts courses. Two formulas for figuring the number of board feet of lumber in a board or project are briefly discussed in the text. There are no examples showing how to use the formulas. There are no problems for the student to use for practice.

Fierer's (2) book is supplemented with a workbook (3) which contains many

good mathematical problems. In fact, each chapter of the workbook has a section devoted to problems. However, students have great difficulty with these problems because of the lack of examples in the text.

Walker's (5) book on metalworking contains several formulas and several good examples of how to use them. It also contains quite a number of problems to give students practice. However, more fundamental instruction is necessary before a student can work these problems.

Summarizing, the examination of textbooks shows that books intended for vocational training are very thorough and begin with instructions in basic fundamentals of arithmetic. These texts, however, are too extensive and the industrial arts teacher would have to use selected portions. This probably would result in lack of continuity in the teaching of shop mathematics, because many industrial arts teachers do not have the proper mathematics background to organize this area of teaching.

The textbooks designed for industrial arts courses contain little or no material on shop mathematics. The books that do contain some mathematics usually assume that students know how to apply the basic fundamentals and how to use a formula. Experience shows that industrial arts students have difficulty in the basic fundamentals of arithmetic. Students have difficulty in the use of fractions which is contained in most shop work. They also have difficulty in the use of formulas. Therefore, a teacher of industrial arts must supplement the text if he is to effectively teach any shop mathematics.

The above discussion indicates that available teaching material for shop

mathematics is not adaptable to use in industrial arts courses. It is, therefore, the purpose of this study to investigate the mathematical needs and abilities of industrial arts students, and based on the results of the investigation, develop a short text suitable for teaching industrial arts shop mathematics.

METHOD OF PROCEDURE

The purpose of this study was to investigate the mathematical abilities and needs of high school industrial arts students. The results of the investigation was used as a guide in developing a short text and handbook on industrial arts mathematics.

Determination of mathematical abilities and needs was accomplished in three ways.

1. A test was prepared and administered to high school industrial arts students in the Des Moines high schools.
2. Central Iowa industrial arts teachers were surveyed by questionnaire to obtain their opinions on student mathematical ability as observed during class activities. Also, opinions were obtained on the desirability of a text for teaching industrial arts mathematics.
3. Supervisory personnel of Des Moines business and industrial firms were surveyed by questionnaire to determine the mathematical capabilities of workers and the areas of difficulty.

Testing High School Students

The mathematics test was administered to industrial arts students in the four Des Moines high schools. A sample copy of the mathematics test is shown in appendix "A". No testing was done at Des Moines Technical High School because it offers no industrial arts courses.

Testing was limited to students enrolled in woodworking and metalworking classes. It was assumed that many students enrolled in industrial arts drafting courses would bias the results of students' mathematical abilities. Many students enrolled

in drafting classes plan to continue their education in the fields of science, mathematics and engineering. These students would have above average aptitudes and abilities in mathematics.

Only those students present on the day the test was administered were tested. No attempt was made to test absentees because of the additional time and inconvenience it would require of the cooperating teachers. Also, it is doubtful the absentee testing would have made any significant difference in the results.

Two woodworking classes and one metalworking class were tested in each of the four high schools. A total of 233 students were tested.

The mathematics test used to determine the ability of high school industrial arts students was composed of thirty-four problems. Its purpose was not only to determine the ability of students, but, to determine their needs for further instruction and practice.

Each test item was included for a definite purpose which will be explained in detail later. An attempt was made to include all of the more common operations required in solving the problems one finds in industrial arts work.

Practical mathematical problems originate in the mind or through discussion with another person. In any event, the mathematical problem must be transferred from thoughts or words into a definite number form. In order to make this transformation, one must be able to place in numeral form a number which is stated in word form. One must also be able to arrange the numbers in the proper way to perform a given mathematical operation. Finally, one must be able to combine the mathematical operations in such a way as to obtain the desired results.

The overall purpose of the test is to determine the ability of students to accomplish the above tasks. Each test item is designed to indicate the weaknesses in performing the separate operations of problem solving.

The following list of statements are identified by the problem number on the test. After each problem number the type of the problem is given followed by the reason or purpose for the problem.

<u>Type of problem</u>	<u>Purpose of problem</u>
1. Translating whole numbers from word form into numeral form.	To determine students understanding of numbers when given verbally or in word form. Also to test his ability to write these numbers in numeral form.
2. Translating decimal numbers given in word form into numeral form.	Same as 1 except for number type.
3. Translating common fraction numbers from word form into numeral form.	Same as 1 except for number type.
4. Adding decimal numbers when problem is already set up.	To determine student's ability to add decimal numbers and properly locate decimal point in the solution.
5. Adding whole numbers when problem is set up.	To determine ability to add correctly.
6. Adding whole numbers when problem is not set up	To determine ability to set up common whole number addition problem correctly.
7. Adding decimal numbers when problem is not set up.	To determine ability to set up addition problems of decimal numbers and to properly locate decimal points.
8. Subtracting whole numbers when problem is set up.	To determine ability to subtract correctly.

<u>Type of problem</u>	<u>Purpose of problem</u>
9. Subtracting decimal numbers when problem is set up.	To determine ability to subtract correctly and properly locate decimal point in solution.
10. Subtracting whole numbers when problem is not set up.	To determine ability to properly set up subtraction problems.
11. Subtracting decimal numbers when problem is not set up.	To determine ability to set up and solve subtraction problems of decimal numbers, properly locating all decimal points.
12. Multiplication of whole numbers.	To determine ability to multiply.
13. Multiplication of decimal numbers.	To determine ability to multiply decimal numbers and properly locate decimal point in the solution.
14. Multiplication of whole numbers when problem is not set up.	To determine ability to set up multiplication problems involving only whole numbers.
15. Multiplication of decimal numbers when problem is not set up.	To determine ability to set up multiplication problems involving decimal numbers.
16. Division of whole numbers.	To determine ability to divide whole numbers.
17. Division of decimal numbers.	To determine ability to properly locate the decimal point.
18. Division of whole numbers when problem is not set up.	To determine ability to set up division problems involving only whole numbers.
19. Division of decimal numbers when problems are not set up.	To determine ability to set up division problems involving decimal numbers, properly locating all decimal points.
20. Converting a common fraction to a decimal fraction.	To determine the student's knowledge of fractions and his ability to convert numbers from one form to another.

- | | |
|--|--|
| 21. Adding common fractions. | To determine ability to add common fractions resulting in a solution which is a common fraction. |
| 22. Adding common fractions, the solution of which is a mixed number. | To determine ability to add common fractions resulting in solutions which are mixed numbers. |
| 23. Adding mixed numbers. | To determine ability to add mixed numbers. |
| 24. Subtracting common fractions. | To determine ability to subtract common fractions resulting in a solution which is a common fraction. |
| 25. Subtracting a common fraction from a whole number. | To determine ability to solve problems involving subtraction of common fractions and whole numbers and resulting in a mixed number solution. |
| 26. Subtracting mixed numbers. | To determine ability to subtract mixed numbers. |
| 27. Multiplication of common fractions. | To determine ability to multiply common fractions resulting in a solution which is a common fraction. |
| 28. Multiplication of common fractions which result in a mixed number. | To determine ability to multiply common fractions resulting in a mixed number solution. |
| 29. Multiplication of mixed numbers. | To determine ability to multiply mixed number. |
| 30. Division of a common fraction by a whole number. | To determine ability to divide a fraction by a whole number. |
| 31. Division of mixed numbers. | To determine ability to divide mixed numbers resulting in a solution which is a mixed number. |
| 32. Solution of a practical problem. | To determine the ability to apply the fundamentals of mathematics to solve a practical problem. |

- | | | |
|-----|---|--|
| 33. | Solving a common problem of woodworking involving the use of a formula. | To determine the ability to use a formula properly and perform the required mathematical operations. |
| 34. | Solving a common electronics problem, using a formula | To determine the use of a fairly complex formula requiring more complex mathematical operations. |

Survey of Industrial Arts Teachers

In the course of designing a project, drawing plans, writing a bill of materials and constructing a project, a student must solve many mathematical problems. The industrial arts teacher spends many hours each semester helping students solve these problems. Because of this relationship with the students, an industrial arts teacher is in an ideal position to evaluate the student's knowledge and the student's ability to apply this knowledge.

A questionnaire was designed to obtain the opinions of industrial arts teachers on the following items:

1. Mathematical ability of students.
2. Types of mathematical problems which are difficult for students.
3. Possible increased student interest to learn mathematics applied to industrial arts work.
4. Need for a text on mathematics designed for industrial arts.
5. Contents of a text for industrial arts mathematics.

A sample of twenty-five industrial arts teachers was selected for this survey. The questionnaire was completed by fourteen industrial arts teachers in the Des Moines high schools and schools of surrounding communities. The remaining eleven were completed by industrial arts teachers attending classes at Iowa State University. A copy of the questionnaire is located in appendix "B".

Survey of Supervisory Personnel in Business and Industry

An assumption was made at the outset of this study that many industrial arts students will be employed in the semi-skilled positions in industry. It was also assumed that many semi-skilled employees in industry are lacking in ability to solve the mathematical problems connected with their work. A third assumption was that there would be a greater desire to learn mathematics if it were taught as applied mathematics in an area of interest; namely, industrial arts.

These assumptions were based on past experience of the writer. This experience includes nine years of work in industry and eight years as a teacher of mathematics and industrial arts.

The questionnaire was designed to obtain opinions on the following points:

1. Ability of employees to solve on-the-job mathematical problems without help.
2. The need for better mathematics background of employees for performance of present work and for advancement to better positions.
3. Possibility of better qualified future personnel if applied mathematics were taught as a unit of high school industrial arts courses.
4. The level of ability of employees in performing the operation of mathematics.

This questionnaire was completed by twenty supervisors of men working in the semi-skilled positions of metalworking and woodworking concerns in the Des Moines area. Appendix "C" is a copy of the questionnaire used.

The supervisors surveyed were in the following businesses:

1. Contractors -- home building.
2. Contractors -- remodeling.
3. Wood products manufacturing.
4. Lumber yards.
5. Metal products manufacturing.
6. Sheetmetal fabrication.
7. Tool and die manufacturing.
8. Machine shops.

FINDINGS

The purpose of this study was to determine the need for, desirability of, and contents of a mathematics text designed for use in high school industrial arts classes. A mathematics text was to be prepared based on the results of the investigation.

Two questionnaires and a mathematics test were developed and used to obtain data from the following sources:

1. High school industrial arts students.
2. High school industrial arts teachers.
3. Supervisors of semi-skilled employees engaged in woodwork and metalwork.

Testing of High School Industrial Arts Students

The test for high school industrial arts students consisted of 34 problems. It was designed to determine the student's ability to perform and apply the most common mathematical operations used in industrial arts work.

The test was administered to 233 students at the four Des Moines high schools. Two woodwork classes and one metalwork class were tested at each school. The distribution of the students by schools and the average percent of incorrect responses for each school is listed in Table 1.

The overall average of incorrect answers was consistent among all schools, except for Roosevelt High School. A possible explanation for the better performance by Roosevelt students is that mathematics was reviewed for two class periods three weeks prior to administering the test.

Table 1. Distribution of students tested and results by schools

School	Number Tested	Average % Incorrect Answers
East	52	43.9
Lincoln	76	46.2
North	62	46.2
Roosevelt	43	36.7

Table 2 shows the total number of students that worked each problem on the test incorrectly. It also shows this number converted to percentage.

Table 2. Results of testing of 233 high school students

Problem No.	Number of incorrect responses	Percentage of incorrect responses
1	20	8.5
2	109	46.7
3	28	12.0
4	45	18.9
5	38	16.3
6	25	10.7
7	47	20.2
8	29	12.5
9	43	18.5
10	69	29.6
11	123	52.8
12	74	31.8
13	106	45.4
14	63	26.6
15	144	61.8
16	30	12.9
17	104	44.6
18	187	80.3
19	174	74.7
20	155	66.5
21	48	20.6
22	91	39.1
23	78	33.4

Table 2 (Continued)

Problem No.	Number of incorrect responses	Percentage of incorrect responses
24	72	30.9
25	66	28.3
26	85	36.5
27	114	48.9
28	156	66.9
29	214	91.9
30	135	57.9
31	198	85.0
32	159	68.2
33	218	93.6
34	233	100.0

The number of incorrectly solved test problems shown in Table 2 indicates a serious lack of ability in mathematics. The seriousness of this lack of ability becomes more apparent upon examination of the test items. Except for the last three items, the test was composed of problems which arithmetic students in sixth grade are expected to solve. The last three items of the test are of seventh grade level of difficulty. The over-all percentage of incorrectly solved problems was 43.9.

In summary, the test results indicate that high school industrial arts students are capable of performing the fundamental operations of addition, subtraction, multiplication and division of whole numbers. The ability of students decreases rapidly when required to set up problems and to perform operations involving decimal numbers and common fractions. Items 32, 33, and 34 of the test results indicate a very severe lack of ability to apply mathematics to a practical situation.

Survey of Industrial Arts Teachers

Twenty-five industrial arts teachers were surveyed by questionnaire to obtain their opinion on mathematical ability of their students. Information was also obtained

relative to interest, need and desired content of a text on industrial arts mathematics.

Eight percent of the teachers surveyed rated their students very low in ability to solve the mathematics problems related to their industrial arts work. Sixty-four percent of the teachers rated their students low in mathematical ability and twenty-eight percent rated them average. None of the teachers rated their students above average in mathematical ability.

The opinion of eighty-four percent of the teachers surveyed indicated student interest in learning mathematics would be greater if mathematics were taught in industrial arts courses.

All teachers surveyed felt that a text on industrial arts mathematics would be a useful aid for instructing industrial arts students. Ninety-six percent of the teachers expressed a need for such a text.

Table 3 shows the results of the survey of industrial arts teachers' opinions on student ability to perform mathematical operations.

Table 3 also shows what industrial arts teachers think should be included in a text on industrial arts mathematics.

Analysis of the results of this survey indicates the following:

1. Industrial arts students are low in mathematical ability.
2. Teachers feel that students would be interested in learning mathematics if taught in relation to industrial arts work.
3. Teachers are interested in obtaining a textbook on industrial arts mathematics.
4. Contents of a mathematics textbook for industrial arts should include

units on basic mathematical operations of addition, subtraction, multiplication and division followed by units in applied mathematics.

Table 3. Teacher opinion of students' mathematical abilities and desired content of mathematics text

Mathematical operation	Teachers indicating operation is difficult for students %	Teachers desiring unit in text %
Addition	16	48
Subtraction	20	48
Multiplication	52	68
Division	52	72
Fractions	100	96
Percentage	80	92
Using a formula	100	88
Algebraic operations	100	80

The opinions of industrial arts teachers, indicated by this survey, are in complete agreement with needs and abilities of students as determined by student test performance.

Survey of Supervisory Personnel

The survey of supervisory personnel was accomplished with the use of a questionnaire. The questionnaire was designed to obtain information on the mathematical ability and needs of semi-skilled employees in the woodworking and metal-working industries. Twenty supervisors completed the questionnaire.

Of the twenty supervisors, seventy-five percent stated that employees were

required to work mathematics problems in connection with their job. Twenty percent of the supervisors indicated that employees were not required to work mathematics problems because the employee lacked mathematical ability.

The survey results show that eighty percent of the employees require some help in working required mathematics problems.

Ninety-five percent of the supervisors surveyed indicated a general need for better mathematical background of employees.

Table 4. Supervisory rating of employees' mathematical ability

Mathematical operation	Capable %	Weak %	Need help %
Addition	80	10	10
Subtraction	70	20	10
Multiplication	60	20	10
Division	40	35	25
Fractions	20	40	40
Percentage	15	45	40
Using a formula	0	30	70
Algebraic operations	0	25	75

Lack of mathematical ability was listed as a common reason for an employee not advancing to a better position, by fifty-five percent of the supervisors surveyed.

One hundred percent of the supervisors stated that a unit of related mathematics in industrial arts courses would better qualify a student for work in industry.

Table 4 shows the percentage of supervisors rating their personnel on ability

to perform the indicated mathematical operations.

The survey of supervisory personnel shows results somewhat similar to the survey of industrial arts teachers and the testing of high school students. Supervisors rate their employees higher, in mathematical ability, than the rating of students given by teachers. This higher rating is to be expected, however, because the employees have had work experience and have received some training on the job.

The findings on this investigation can be summarized by the following statements:

1. High school industrial arts students are very low in mathematical ability.
2. Semi-skilled employees in industry are lacking in mathematical ability.
3. Industrial arts teachers feel that students would benefit from a unit on mathematics related to the activities of industrial arts courses.
4. Industrial arts teachers desire a mathematics text designed for industrial arts.
5. Industrial supervisors feel that applied mathematics taught in industrial arts courses would better qualify students for industrial employment.
6. The contents of a text for teaching industrial arts mathematics should include:
 - a. Addition
 - b. Subtraction
 - c. Multiplication
 - d. Division
 - e. Fractions
 - f. Formulas
 - g. Algebra

DISCUSSION

In recent years, great changes have occurred in the teaching of mathematics. These changes were brought about by the rapid scientific and technological growth of our society. Mathematics is an integral part of science and technology and has, therefore, experience similar growth.

Several mathematics committees have been established in recent years to study teaching methods and textbook content. Their purpose was to find a way of satisfying the greater demands placed on mathematical knowledge. Some of the committees developed textbooks which differed considerably in method of presentation and content from the conventional mathematics textbooks. Nationwide experimental trial of these textbooks proved them acceptable and the modern mathematics program was established throughout the nation.

Modern mathematics places greater emphasis on learning the theory of mathematical operations. The purpose is that a person with a good theoretical background will be able to apply the theory to the solution of practical mathematics problems.

Conventional mathematics places greater emphasis on learning by drill, memorizing rules and practice on applied problems from various fields.

The theoretical approach to learning mathematics is useful and necessary for students planning a career requiring higher mathematics. But, many students lack interest in mathematics unless they can see its value in direct application to an area of their interest.

Conventional mathematics textbooks usually have several chapters devoted

to applied problems in specific areas. The purpose of these chapters is to show the importance of mathematics in various occupations and to create interest in mathematics by relating it to practical life situations. The problem of maintaining student interest and desire to learn still remains, because all students are not interested in all of the areas of application.

There is no doubt that modern mathematics will be beneficial to students who have the required interest and ability to advance to higher levels of mathematical knowledge. However, the writer feels that students of low mathematical ability and interest will show a decline in mathematical learning.

The end result of this study is the preparation of a textbook on mathematics for use in teaching mathematics in industrial arts classes. One might question the idea of teaching mathematics in an industrial arts class when students are required to take courses for several years which are specifically devoted to mathematics.

Partial justification for teaching mathematics in industrial arts classes is implied in the discussion of modern mathematics. Modern mathematics is theoretical whereas industrial arts mathematics is applied and practical. Mathematics taught in industrial arts classes is directly applied to work of great interest to the student.

The findings of this study show industrial arts students are very low in mathematical ability. Most of them have completed the required courses in mathematics and still are unable to work the problems connected with their industrial arts activities. The need for mathematical knowledge is evident and instruction should be given.

The interest shown by the people contacted during this study was surprising,

as well as very encouraging. Of all the industrial arts teachers contacted, only one indicated no interest in the proposed mathematics text. He was of the opinion that teaching mathematics was a responsibility of mathematics teachers. All other teachers indicated strong interest in the study.

An interview with the owner of a tool and die manufacturing company led to some comments worthy of discussion. He was very enthusiastic about this study because of personal experience and mathematical inabilities of his employees.

This person stated that mathematics was the most difficult subject he took in high school. He didn't understand what mathematics was all about, he had no interest in mathematics and his accomplishments were barely enough to pass the course. He claimed he got absolutely nothing out of his mathematics classes. After graduation from high school, he decided to become a machinist. In trade school, he had a three-month course in machine shop mathematics. Because this course was applied to machine shop work, he claims it was one of the most interesting course he ever had. He could see a need for mathematics and became interested.

Another incident indicating a desire to learn mathematics when related to an area of interest or need involves a particular job performed in the tool and die shop. The set up and machining on this job require several complex calculations. The owner claimed he loses money every time the job is performed because all the employees stop work to watch and learn the mathematics of the job. He commented his employees were weak on mathematical ability and that most job applicants also have this weakness.

The manager of a lumber yard was very emphatic in his opinion that related mathematics taught in industrial arts courses would result in more qualified employees. He ranked mathematical ability and poor penmanship as the two most frequent reasons for not hiring job applicants.

The writer witnessed many similar displays of interest in mathematics while employed in industry. The writer has nine years' experience in industry, eight years' experience teaching mathematics and four years' experience teaching industrial arts. This experience permitted the writer to observe many individuals in the light of this study. A very close relationship between mathematics, industrial arts, became apparent over the years. It was this relationship that prompted the selection of this study.

The text developed as a result of this study was quite different in content and design than was expected at the out-set of the study. The findings indicated a need for strengthening student knowledge of basic fundamentals of arithmetic. Some explanation is needed on how this text applies to industrial arts mathematics.

It is suggested that a class be given a diagnostic test similar to the one used for this study. In this way, the needs of the class can be established. Reading assignments and discussions can be determined by the results of the test, either on a class basis or on an individual basis.

An attempt was made to be very thorough in explaining why we perform basic arithmetic operations the way we do. Many high school students have trouble with mathematics because they cannot remember all the rules. It is quite possible that improvements will result if the students are given reasons instead of rules.

Starting with the unit on formulas, the rest of the text is general in nature. Discussion of the material in the text, supplemented with teacher-directed application to specify industrial arts problems will adapt the text to teaching mathematics to any industrial arts class.

INDUSTRIAL ARTS MATHEMATICS

INTRODUCTION

Industrial arts is too often thought of as shopwork, where students make projects. This is thought to be the extent of the subject. In reality, the project is only an interesting method used to familiarize the students with hand tools, power machinery, materials and processes. Also, the project gives the student experience in design, organization of procedures and the satisfaction of creating some useful object.

In the process of the construction of a project, the student is confronted with many opportunities to apply his knowledge of mathematics. Unfortunately, many students find they have insufficient ability to solve the necessary mathematical problems.

The textbooks used for industrial arts courses generally contain very little, if any, instruction in mathematics related to the course. Most of these books contain a few formulas, but no instruction or practice exercises on their use.

There are many good mathematics textbooks for the various vocational areas, but they are too involved for the needs of the industrial arts students.

This text was developed to fill the needs of the industrial arts student. It briefly covers the basic fundamentals of arithmetic and explains the operations. It also includes short sections on algebra, geometry and trigonometry. Only information useful for solving practical problems in industrial arts is included.

This text is not meant for use in teaching a course in mathematics, but rather as an aid in teaching the mathematics necessary for a given industrial arts course. This will vary from one class to another. Sections of the text may be

selected as needed.

The student should consider this text as being a manual of industrial arts mathematics. It should be kept as a permanent reference for use whenever the student discovers his knowledge is lacking in attempting to solve a problem.

The material covered in this text was selected on the basis of results of an extensive investigation.

1. High school industrial arts students were tested to determine their ability to solve industrial arts problems.
2. Supervisory personnel of business and industry were surveyed to determine thoughts on the mathematical ability and needs of workers.
3. Industrial arts teachers were surveyed to obtain their impressions of student needs in mathematics.

PREFACE

Successful problem solving requires:

1. An understanding of numbers.
2. Skill in accurate computation.
3. An understanding of the operations of arithmetic.
4. An understanding of the language of mathematics.
5. The ability to use the proper operations in the proper sequence to obtain a correct solution.

The above statements indicate that problem solving, first of all, demands a thorough understanding of numbers and arithmetic operations. We will, therefore, begin with reading, writing and understanding numbers and then review all the operations we will perform with numbers.

Keep in mind that problem solving requires a systematic, organized approach.

Steps in problem solving:

1. Study the problem until it is thoroughly understood.
2. Decide which mathematical operations are required.
3. Establish the sequence of steps leading to the solution of the problem.
4. Make an estimate of the solution.
5. Perform the necessary computations.
6. Compare calculated solution to the estimated solution.
7. If the two solutions are not reasonably close, check for errors.

WHOLE NUMBERS

Our number system is made up of the ten digits, 1 2 3 4 5 6 7 8 9 0. With these ten digits and the use of place-value, it is possible to write or state any number. It is important that one have a thorough understanding of whole numbers, digits and place-value in order to communicate with others and to solve mathematical problems.

Each digit has a definite value, depending on where it is placed in a number. This is the idea of a place-value.

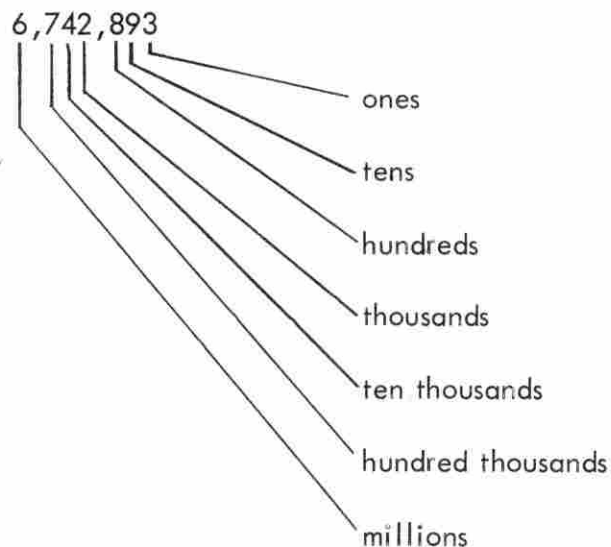
1 - the place-value of 1 is one.

10 - the place-value of 1 is ten.

100 - the place-value of 1 is a hundred.

1,000 - the place-value of 1 is a thousand.

In other words, as a digit moves to the left one place, its value becomes ten times as great. The following example shows the name of each place up to millions:



When saying or writing a number, the name of each place is not used.

Notice that a number has a comma after each third digit as one moves from right to left. These commas are for convenience in reading the number. Except for the first three digits on the right, a number is read by reading from the left the number up to the first comma and give it the name of the place-value just to the left of the comma. Then read the number up to the next comma and give it the name of the place-value of the digit to the left of the comma. This process continues until all digits have been read. The last three digits on the right are read as any number between one and nine hundred ninety nine.

The number in the example above would be read six million seven hundred forty two thousand eight hundred ninety three.

The number, eighty four million seven hundred thirty seven thousand three hundred two would be written 84,737,302.

As will be seen, place-value plays an important part in establishing the methods used in performing the basic operations of arithmetic.

Addition of whole numbers

When adding whole numbers, only digits having the same place-value can be combined. This determines the method of setting up the problem. To illustrate, we will add the number 1,628 and 732.

Correct

$$\begin{array}{r} 1,628 \\ 732 \\ \hline 2,360 \end{array}$$

Incorrect

$$\begin{array}{r} 1,628 \\ 732 \\ \hline 8,948 \end{array}$$

The incorrect solution, on the right, is fairly common. If we think of

place-value in adding whole numbers, we can quickly see why the method is incorrect. Notice that we added the digit 3 in one number to the digit 6 in the other. The 3 has a place-value of tens, whereas the 6 has a place-value of hundreds. What would we call the result 9? In the answer, we call it hundreds. This is obviously false.

We are then led to the correct method of adding whole numbers. Add together only digits having the same place-value. In adding whole numbers, the problem will always be set up correctly if the right-hand digits or the ones digits are lined up in a vertical column. All other digits will automatically line up in proper position.

Notice, when the ones digits are added, we have ten ones. As was mentioned before, as we move to the left, each digit has a place-value ten times as great. Therefore, 8 ones plus 2 ones gives 10 ones. Ten ones makes one ten so we place the 1 in the tens place, adding it to the sum of the 2 and 3. The same thing takes place as we move from right to left in adding the columns.

To add whole numbers, set up the numbers so the digits of like place-value are in vertical columns. Proceed to add each column beginning at the right and add multiples of ten for each column to the next column to the left.

Add 261, 1,473 and 25,628:

$$\begin{array}{r} 25,628 \\ 1,473 \\ 261 \\ \hline 27,362 \end{array}$$

or

$$\begin{array}{r} 1,473 \\ 25,628 \\ 261 \\ \hline 27,362 \end{array}$$

The order in which whole numbers are added is unimportant and will not

change the result.

Addition is indicated by the words add to, plus, sum, more than, increased and by the symbol $+$.

Subtraction of whole numbers

Subtracting whole numbers requires a set up of the problem similar to addition. There are, however, two important differences.

1. Subtracting can be accomplished with only two numbers at one time.
2. The order of setting up the numbers is very important.

Subtract 678 from 1,204. The proper way to set up this problem is as follows:

$$\begin{array}{r} 1,204 \\ - 678 \\ \hline 526 \end{array}$$

position of number to be subtracted

As in addition, only numbers of like place-value can be subtracted. In the ones column, it will be noticed that 8 cannot be subtracted from 4. But, remember that the next digit to the left of the 4 has a place-value ten times as great. If we take one unit from the digit to the left, it will have a value of 10 ones plus the 4 we have making 14. Subtracting the 8 from 14 gives us the answer of 6. This procedure can be used as required in subtracting one column at a time starting on the right and moving to the left.

To subtract whole numbers, set up the two numbers one above the other with digits of like place-value in line. Also, place the number to be subtracted below the other. Proceed to subtract one place-value column at a time, beginning on the right. Borrow a unit from the digit to the left when necessary.

Subtract 467 from 3,428:

$$\begin{array}{r} 3,428 \\ - 467 \\ \hline 2,961 \end{array}$$

Subtraction is indicated by the words, minus, less than, difference, diminished, decreased and the symbol $-$.

Multiplication of whole numbers

Multiplication of whole numbers is somewhat similar to addition. In fact, multiplication is a rapid method of adding the same number many times. Multiplying 21 by 3, is the same as adding $21 + 21 + 21$.

In multiplication, like in addition, the order of multiplication is unimportant. However, there is a difference in the meaning. To multiply 21 by 3 means the same as adding three of the number 21. To multiply 3 by 21 means the same as adding twenty-one of the number 3. The result, however, will be the same.

Like the other operations of arithmetic, the method of performing the operation of multiplication is determined by the concept of place-value. To illustrate, we will multiply 347 by 213. Usually, this problem is set up as shown, but this set up developed because of a desire for standard methods rather than because of necessity.

$$\begin{array}{r} 347 \\ \times 213 \\ \hline \end{array}$$

The problem means that we determine how many units of each place-value we will have as a result of multiplying the two numbers. The 3 is in the ones place, so we will have 3 times 347 as the number of ones. The 1 is in the tens place, so we will have 1 times 347 in the tens place. The 2 is in the hundreds place, so we

will have 2 times 347 in the hundreds place.

$$\begin{array}{r}
 347 \\
 \times 213 \\
 \hline
 1041 \\
 347 \\
 694 \\
 \hline
 73911
 \end{array}$$

1041 ones
 347 tens
 694 hundreds

The tens and hundreds can be changed to ones by adding zeros to the right end of the number until the ones place is reached. For instance, adding two zeros to 694 hundreds would give us 69400 ones. In effect, this is what we do when we add up the ones, tens, and hundreds to get our final answer. The column of digits on the right contains only the digit 1 and this is what we have in our answer. The same answer would result if we made the above mentioned changes. The problem would look like this:

$$\begin{array}{r}
 347 \\
 \times 213 \\
 \hline
 1041 \\
 3470 \\
 69400 \\
 \hline
 73911
 \end{array}$$

But, the same result is obtained by leaving the zeros out and this is usually done.

As mentioned before, the problem could be set up differently, but the first method has become standard. The important thing is to understand what you are doing.

The problem could be set up in the following way:

$$\begin{array}{r}
 347 \\
 \times 213 \\
 \hline
 \end{array}$$

However, this set up makes it more difficult to properly locate all numbers in relation to their place-value.

The first method shown has the advantage of having all ones digits in the ones column, all tens digits in the tens column, etc. Therefore, it is the method which is most desirable.

To multiply two whole numbers, set up the problem by placing one number beneath the other with digits of like place-value lined up vertically. Then proceed to multiply one number by each digit of the other number, placing the partial answers in proper location according to their place-value. Then add the partial answers to obtain the final result.

Multiply 4,672 by 173.

$$\begin{array}{r}
 4672 \\
 \times 173 \\
 \hline
 14016 \quad) \\
 32704 \quad) \text{ partial answers} \\
 4672 \quad) \\
 \hline
 808256
 \end{array}$$

To multiply by a number containing zeros sometimes causes difficulty. If you remember place-value, you can avoid this difficulty. This means there will be no number in the row for the tens number.

$$\begin{array}{r}
 1473 \\
 \times 602 \\
 \hline
 2946 \\
 0000 \\
 8838 \\
 \hline
 \end{array}$$

Normally, instead of writing the whole row of zeros, only the one on the right is used and then the lower row of digits is moved up to replace the other

zeros so the problem would look like this:

$$\begin{array}{r} 1473 \\ \times 602 \\ \hline 2946 \\ 88380 \\ \hline 886746 \end{array}$$

Notice that the digits 8838 are still in the same position, left to right.

Multiplication is indicated by the words, product, times, and factor. It is also indicated by the symbol \times , by the use of parenthesis and by the use of a dot.

Examples indicating multiplication:

$$3 \times 24 =$$

$$3 (24) =$$

$$3 \cdot 24 =$$

Division of whole numbers

The same similarity exists between subtraction and division as exists between addition and multiplication.

Multiplication is a rapid method of adding the same number many times; division is a rapid method of determining how many times the same number can be subtracted from another number. The problem 6 divided by 2 gives an answer of 3. This means you can subtract 2 from 6 three times.

Place-value, as in the other operations plays an important role in determining the proper method of performing the operation of division of whole numbers. To illustrate, we will divide 12 into 3,624. The problem is set up as shown:

$$12 \overline{) 3624}$$

The number 12 cannot be divided into the digit 3 of the number 3,624. The digit 3 in this number has a place-value of thousands. The fact that the digit 3 cannot be divided by 12 means that 12 will go into the number 3,624 less than

a thousand times, so there will be no thousands digit in the answer. This can be shown by placing a zero in the thousands place in the answer; that is, directly over the 3.

$$\begin{array}{r} 0 \\ 12 \overline{) 3624} \end{array}$$

Next, determine if the 12 can be divided into the first two digits of 3,624. The first two digits of 3,624 are 36 and mean 36 hundreds because the 6 is in the hundreds place. The number 12 will go into the number 36 three times. Therefore, we place a 3 in the hundreds place, meaning 12 will go into 3,624 at least three hundred times.

$$\begin{array}{r} 03 \\ 12 \overline{) 3624} \end{array}$$

Multiplying 3×12 , we get 36, meaning 36 hundreds. Thus, we must subtract 36 hundreds from 3,624. This is done by placing the 36 beneath the 3,624 according to place-value and proceeding to subtract.

$$\begin{array}{r} 03 \\ 12 \overline{) 3624} \\ \underline{36} \\ 0 \end{array}$$

There remains only 24 from the original number 3,624:

$$\begin{array}{r} 03 \\ 12 \overline{) 3624} \\ \underline{36} \\ 024 \end{array}$$

Remember that 36 hundreds is the same as 3,600 ones; therefore, this step could be written as shown.

41

$$\begin{array}{r} 03 \\ 12 \overline{) 3624} \\ \underline{3600} \\ 024 \end{array}$$

The extra zeros, however, are eliminated to shorten the process. The zeros are just understood to exist.

$$\begin{array}{r} 3 \\ 12 \overline{) 3624} \\ \underline{36} \\ 24 - \text{remainder} \end{array}$$

The 2 in the remainder 24 is in the tens place. Therefore, to determine a digit for the tens place in the answer you find how many times the 12 will go into the 2. It will not go into the 2, so a zero is placed in the tens place in the answer.

$$\begin{array}{r} 30 \\ 12 \overline{) 3624} \\ \underline{36} \\ 24 \end{array}$$

Finally, it is found that 12 will go into the 24 ones a total of 2 times. The 2 is then placed in the ones place in the answer.

$$\begin{array}{r} 302 \\ 12 \overline{) 3624} \\ \underline{36} \\ 24 \\ \underline{24} \end{array}$$

Multiplying 2×12 , we get 24 to subtract.

$$\begin{array}{r} 302 \\ 12 \overline{) 3624} \\ \underline{36} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

This leaves no remainder. If there were a remainder, it would be listed with the answer as a remainder or as a fraction.

A problem will now be shown which has a remainder:

$$\begin{array}{r} 29 \\ 16 \overline{) 472} \\ \underline{32} \\ 152 \\ \underline{144} \\ 8 \end{array}$$

The remainder would be shown in the answer as 29 R 8 or $29 \frac{1}{2}$. The $\frac{1}{2}$ is obtained by placing the 8 over the 16 in fraction form $\frac{8}{16}$, which reduces to $\frac{1}{2}$.

Division is indicated by the words, what part of, quotient, goes into, and by the symbols $\overline{)}$, \div , $/$, $-$.

DECIMAL NUMBERS

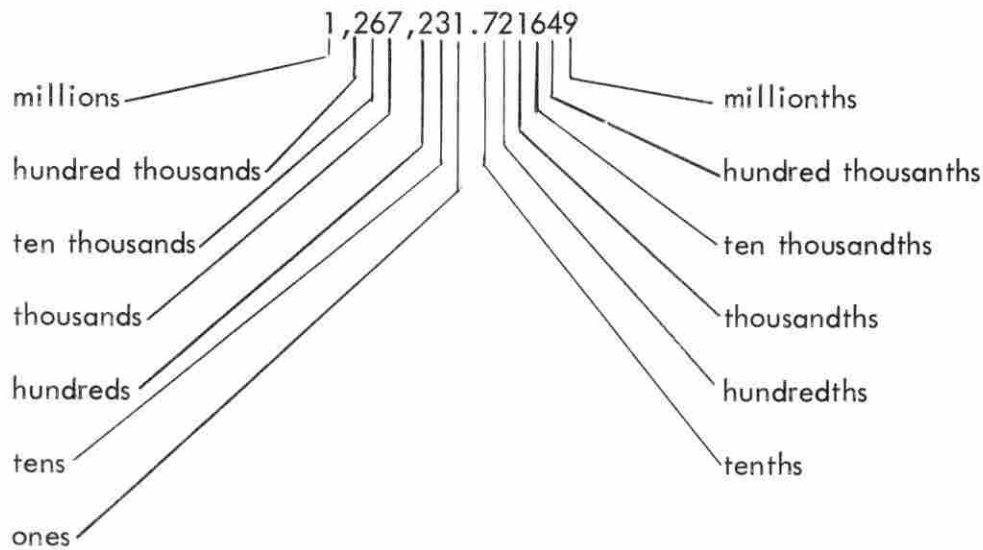
Before discussing the operations of arithmetic on decimal numbers, it will be necessary to expand our understanding of place-value.

Earlier, we had an illustration showing place-values for the digits in a whole number. Any size number, one or greater, can be written as a whole number. But, sometimes we deal with numbers less than one or numbers that have a value between two whole numbers. This means we need a way to express that number or portion of a number less than one.

When we started at the right hand digit of a whole number, we found each digit, as we moved to the left, had a value ten times as great as the one before it. This will continue to be true for decimal numbers.

The illustration following shows that when we start with the ones digit and move to the left, we have names of place-values which are similar to those digits

to the right. The only difference is that names to the left end with 's' while those to the right end with 'ths'. This is the way you can distinguish between whole numbers and decimal numbers when stated verbally or written as word numbers.



Reading a decimal number is different than reading a whole number. The part of a number to the right of a decimal point is read as though it were a whole number and then given the name of the place-value of the last digit. To illustrate this, the number 0.1763 reads, one thousand seven hundred sixty three ten-thousandths.

Another point of importance in reading decimal numbers is the decimal point. If the number contains only the decimal part, there is no problem. But, if the number contains both parts, the whole number and the decimal part, we must have a way of combining the two parts in words.

To read a decimal number containing a whole number part and a decimal part, we use the word "and" to indicate a decimal point. For example, 342.194 would be read three hundred forty two and one hundred ninety four thousandths.

Nineteen ten thousandths would be written as 0.0019. Notice that the 19 is placed so the last digit is in the ten thousandths place and zeros were used to fill spaces up to the decimal place.

Addition of decimal numbers

The procedure for adding decimal numbers is exactly the same as for adding whole numbers except that the decimal point must be properly located.

To add decimal numbers, set up the problem with digits of equal place-value lined up in vertical columns. When this is done, the decimal point will also line up in a vertical column. This position then will be the same in the answer.

Add 6.742, 79.6 and 143.7834.

$$\begin{array}{r} 143.7834 \\ 79.6 \\ 6.742 \\ \hline 230.1254 \end{array}$$

Remember, the numbers can be written in any order in addition problems.

Subtraction of decimal numbers

Like addition, the procedure is the same for subtracting decimal numbers as it is for subtracting whole numbers, except for the decimal point. The decimal point is treated the same as in addition.

Subtract 6.791 from 84.6432.

$$\begin{array}{r} 84.6432 \\ - 6.791 \\ \hline 77.8522 \end{array}$$

Multiplication of decimal numbers

Problems of multiplication of decimal numbers are set up and worked

exactly like whole number problems.

Multiply 41.23 by 3.7.

$$\begin{array}{r} 41.23 \\ \times 3.7 \\ \hline 28861 \\ 12369 \\ \hline 152.551 \end{array}$$

Notice the decimal point in the solution of the above problem. The decimal point in the answer is located by counting the number of digits to the right of the decimal point in the numbers being multiplied and placing this combined number of digits to the right of the decimal point in the answer. There are two digits to the right of the decimal point in 41.23 and one in 3.7; therefore, there must be three digits to the right of the decimal point in the answer.

To explain the location of the decimal point in the answer to multiplication problems, we must examine place-value of the digits involved. Reference to the illustration showing place-value of digits, shown earlier, will be helpful.

As was stated before, each digit in a number has a place-value ten times greater than the one to the right of it. The reverse is also true. A digit has a place-value ten times less than the one to its left.

$$\begin{array}{r} 1.42 \\ \times .6 \\ \hline .852 \end{array}$$

Observe the first operation in the above multiplication problem is to multiply 6 times 2. The 2 has a place-value of hundredths. We multiply the 2 hundredths by the number 6 which has a place-value of tenths meaning that the result must have a place-value ten times less than the place-value of the digit being multiplied.

The place-value of the 2, the number being multiplied, is hundredths. Referring to the illustration showing place-values, it will be seen that the digit to the right of hundredths is thousandths. A thousandth digit has a place-value ten times less than a hundredths digit. Therefore, the answer to the above problem must end with a digit in the thousandths place.

If several problems are solved, it will soon be apparent that the number of digits to the right of the decimal point in the answer is always equal to the sum of the digits to the right of decimal points of the two numbers being multiplied.

Division of decimal numbers

Locating the decimal point in the answer to division problems of decimal numbers can be made more understandable if the divisor is changed to a whole number by moving the decimal point to the extreme right of the number.

Divide .03 into 6.96.

$$\begin{array}{r} 232 \\ .03 \overline{) 6.96} \end{array}$$

Notice, that to make .03 a whole number, the decimal point is moved two places to the right. To keep the relationship of the numbers the same as before, the decimal point in 6.96 must also be moved two places to the right or 696.

The moving of the decimal points can be justified more clearly if we write the above problem in a different form. The form $\frac{6.96}{.03}$ has the same meaning as $.03 \overline{) 6.96}$.

$$6.96 \text{ is the same as } 6.96 \times 100 = 696$$

$$.03 \text{ is the same as } .03 \times 100 = 3$$

Then:

$$\frac{6.96 \times 100}{.03 \times 100} = \frac{696}{3} \text{ is equivalent to } \begin{array}{r} .03 \overline{) 6.96} \end{array}$$

The multiplication of both numbers by 100, is the same as multiplying by 1.

Any number divided by itself is one:

$$\frac{100}{100} = 1; \text{ therefore, } \frac{6.96 \times 100}{.03 \times 100} \text{ is the same as } \frac{6.96}{.03} \times 1.$$

Therefore, if we multiply both numbers of a division problem by the same number, we are multiplying by the number one and the value of the problem does not change.

To divide decimal numbers, move the decimal point in both numbers to the right as many places as required to make the divisor a whole number. Then proceed to divide as in division of whole numbers. The decimal point in the answer will always appear directly above its location in the problem.

$$\begin{array}{r} 0.20 \\ .62 \overline{) 12.40} \end{array}$$

COMMON FRACTIONS

An understanding of common fractions and the operation of arithmetic with fractions is very important to students of industrial arts. One encounters fractions at the very outset of work as the raw materials of industrial arts work have fractional dimensions. For instance, one inch lumber is actually $13/16$ inches thick. A standard six-inch board will actually measure only $5 \frac{5}{8}$ inches. Many drill bits, bolts and screws are of fractional sizes.

It is apparent that you will be involved in many fraction problems in selecting materials, cutting material to size, laying out joints, using tools and in many other

ways.

Before discussing operations with fractions, we must have a definition of them. We must know what a fraction is. We must also explain some terms which we will use in discussing fractions.

A fraction is a part of a whole unit.

The denominator is the number which appears below the line in a fraction. It tells two things. One, it tells the number of parts into which the whole has been divided. Second, it names the fraction. In the fraction $\frac{2}{3}$, the denominator 3 tells us the whole has been divided into three equal parts and also that the name of the fraction is thirds. The fraction $\frac{3}{8}$ would be called three-eighths.

The numerator of a fraction is the number appearing above the line. It shows how many of the equal parts indicated by the denominator that the fraction is describing.

$\frac{3}{4}$ indicates the whole has been divided into four equal parts and that three of these equal parts are being considered.

$\frac{5}{8}$ inches means that an inch has been divided into eight equal parts and that five of these equal parts have been used to describe the desired length.

A proper fraction is a fraction having a value less than one whole. That is, the numerator is less than the denominator. Remember, the symbol $/$ indicates division. So, if the numerator is greater than the denominator, the division would give an answer greater than one whole.

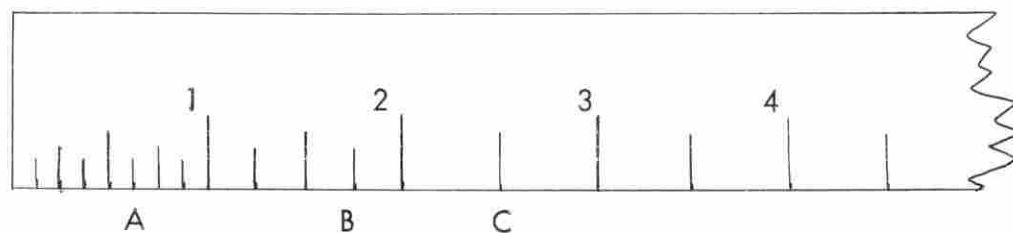
$\frac{6}{2}$ is an improper fraction because if divided by two it is equal to three which is greater than one whole.

A mixed number is a number which is part whole number and part fraction, such as $3 \frac{5}{8}$.

A complex fraction is a fraction which has a fraction or a mixed number as a numerator or denominator or both, such as $\frac{1 \frac{3}{8}}{4}$.

Because measuring is an important skill in industrial arts, we will use the ruler to explain fractions. It must be remembered, though, that fractions apply to anything, including numbers, where units smaller than one whole are considered.

A ruler has the space between the whole inches divided into several equal spaces by marks.



If we consider an inch divided into two equal parts as between 2 and 3 in the illustration, each part of this inch would be $\frac{1}{2}$.

$$\frac{1}{2} \quad \begin{array}{l} \text{one of the equal parts} \\ \text{number of equal parts into which the inch is divided} \end{array}$$

The inch between 1 and 2 is divided into four equal parts so 4 becomes the denominator of the fraction. One of the parts is $\frac{1}{4}$. Two of the parts is $\frac{2}{4}$.

The first inch is divided into eight equal parts, so 8 becomes the denominator of a fraction describing this size. One of the parts is $\frac{1}{8}$. Three of the parts is $\frac{3}{8}$.

The number representing point 'A' on the ruler would be $\frac{5}{8}$, read

five-eighths.

The number representing point 'B' on the ruler would be $1\frac{3}{4}$, read one and three-fourths.

The number representing point 'C' on the ruler would be $2\frac{1}{2}$ read two and one-half.

Notice the word 'and' is used to separate the whole number and the fraction when reading a mixed number.

Addition of fractions

To add means to combine two or more quantities to determine a total quantity. To add fractions then, means to find the total number of parts. The number of parts is indicated only by the numerator or the top number of the fraction. The denominator only indicates the size of the parts. Therefore, to add fractions, only the numerators are combined.

Add $\frac{1}{8}$ and $\frac{2}{8}$.

$$\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$$

indicates how many
names the fraction and indicates the size of its parts

As always, in addition problems, only units of the same value can be added. We cannot add fractions which have different denominators. The fractions can be added, if we change them so they have the same denominator.

To add $\frac{3}{8}$ and $\frac{1}{4}$, the denominators must be the same, so one or both will have to be changed. Looking at the illustration of the ruler, it will be seen that each $\frac{1}{4}$ can be divided into two parts, making eight parts altogether. Each part would then be $\frac{1}{8}$. Two $\frac{1}{8}$ parts is equal to one $\frac{1}{4}$ part. So $\frac{1}{4}$ is the same

as $2/8$.

Now we can write the problem and add:

$$\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

Changing fractions so they have the same denominators is called finding the common denominator. A common denominator is a number which can be divided evenly by all denominators. In other words, the common denominator will be a multiple of all denominators in the problem.

An easy way to find a common denominator is to try multiples of the largest denominator in the problem, starting with two times, then three times, and so on until the first multiple is found which can be divided by all denominators.

If the denominators are 2, 3 and 4, take multiples of the number 4.

$2 \times 4 = 8$; 8 can be divided by 2 and 4 but not by 3.

$3 \times 4 = 12$; 12 can be divided by 2, 3 and 4, so 12 is the denominator to use.

Try again with the numbers 2, 6 and 8.

$2 \times 6 = 12$; 12 is divisible by 2 and 6 but not by 8.

$3 \times 6 = 18$; 18 is divisible by 2 and 6 but not by 8.

$4 \times 6 = 24$; 24 is divisible by 2, 6 and 8 and is therefore, the common denominator.

If we have the problem $1/2 + 4/6 + 3/8$, the above example showed the common denominator to be 24. We have to change all fractions in the problem so they have a denominator of 24. To change $1/2$ to a fraction with a denominator

of 24, determine how many times greater 24 is than 2. It is found to be 12 times as great, so we have to make the numerator 12 times as great. The fraction then becomes $12/24$.

$$\frac{1 \times 12}{2 \times 12} = \frac{12}{24}$$

Changing the other two fractions, we get

$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24} \quad \text{and} \quad \frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

Notice that each time the fraction is multiplied by a fraction whose value is 1, so the value of the fraction is not changed.

$$\frac{12}{12} = 1; \quad \frac{4}{4} = 1; \quad \frac{3}{3} = 1$$

To add fractions, change them to fractions with common denominators, then add as before:

$$1/2 + 5/6 + 3/8 = 12/24 + 20/24 + 9/24 = 41/24$$

The answer above $41/24$, is not a proper fraction, so it must be changed. It takes $24/24$ to make one whole. If we take $24/24$ and call it 1, we will have $17/24$ left. Putting these together, we have a mixed number, $1 \frac{17}{24}$.

Sometimes, the answer to a problem will have a numerator and denominator which are multiples of the same number, such as the fraction $6/9$. The 6 and the 9 are both multiples of 3, meaning both numbers can be divided by 3, and the fraction would then be $2/3$. This should always be done. This is called reducing to lowest terms.

To have a fractional number in proper form means it must be checked

for two things.

1. If the fraction is an improper fraction, change it to a mixed number by dividing the denominator into the numerator and expressing the remainder as a fraction:

$$19/4 = 4 \overline{) 19} \begin{array}{r} 4 \\ 16 \\ \hline 3 \end{array} = 4 \frac{3}{4}$$

3, remainder

2. If the fraction can be reduced, it should be done by dividing the denominator and numerator by the same number:

$$16/28 = \frac{16 \div 4}{28 \div 4} = 4/7$$

Subtraction of common fractions

Subtraction of common fractions requires the same procedures used in addition.

Subtract $3/8$ from $7/8$:

$$7/8 - 3/8 = 4/8 = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

Only the numerators are subtracted. The denominator stays the same as it only names the fraction and indicates the size of the fractional units.

In addition to changing the fractions to common denominators, there is another change that is sometimes necessary in subtraction problems. We can illustrate this change with another problem.

Subtract $7/8$ from $1 \frac{1}{4}$.

We must first change to a common denominator:

$$1 \frac{1}{4} - 7/8 = 1 \frac{2}{8} - 7/8$$

The problem now indicates subtraction of $7/8$ from $2/8$, which cannot be done. However, we have a whole number 1, which is equal to $8/8$. If we change 1 to $8/8$ and add it to the $2/8$, we will have $10/8$ altogether and can then subtract.

$$1 \frac{2}{8} - \frac{7}{8} = \frac{10}{8} - \frac{7}{8} = \frac{3}{8}$$

Subtract $3 \frac{3}{4}$ from $7 \frac{5}{16}$:

$$7 \frac{5}{16} - 3 \frac{3}{4} = 7 \frac{5}{16} - 3 \frac{12}{16} = 6 \frac{21}{16} - 3 \frac{12}{16} = 3 \frac{9}{16}$$

Notice, there are two subtraction operations in the example above. The fractions are subtracted in one operation and the whole numbers are subtracted in another operation.

Multiplication of common fractions

To multiply fractions, the numerators are multiplied and the denominators are multiplied.

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

The method of multiplying fractions is determined by the meaning of the terms of a fraction which we discussed earlier. The denominator of a fraction indicates the number of equal parts into which a unit is divided. The numerator indicates the number of these equal parts.

Looking again at the problem $\frac{2}{3} \times \frac{4}{5}$ we see that we have the fraction $\frac{4}{5}$ to be multiplied by $\frac{2}{3}$. The fraction $\frac{4}{5}$ means that we have 4 of the parts of one whole which has been divided into 5 equal parts.

To multiply $\frac{4}{5}$ by $\frac{2}{3}$ means we must divide each unit of $\frac{4}{5}$, or each $\frac{1}{5}$ into three equal parts. The unit $\frac{1}{5}$ means that one whole has been divided into five equal parts. If we divide each one of these five parts into three equal parts,

the one whole will be divided into fifteen equal parts. That is what happens when we multiply the denominators.

$2/3 \times 4/5 = 8/15$; 8 is the quantity of new parts; 15 is the size of the new parts.

Many times, it is possible to simplify a multiplication process by an operation called cancellation. This operation is similar to reducing fractions.

$$2/3 \times 3/5 = \frac{2 \times 3}{3 \times 5} = \frac{3 \times 2}{3 \times 5} = 3/3 \times 2/5 = 1 \times 2/5 = 2/5$$

In this problem, we see that both numerator and denominator have a factor 3. Rearranging the numbers as shown in the successive steps of solving the above problem we end up with $3/3$ which is equal to 1.

Therefore, whenever the numerator and denominator have factors which are equal, we can divide them out. Two examples will illustrate this operation called cancellation.

$$3/4 \times 8/9 = \overset{1}{\cancel{3}}/\overset{2}{\cancel{4}} \times \overset{4}{\cancel{8}}/\overset{3}{\cancel{9}} = 2/3$$

$$\overset{3}{\cancel{9}}/\overset{1}{\cancel{3}} \times \overset{3}{\cancel{2}}/\overset{1}{\cancel{2}} = 3/8 \times 1/1 = 3/8$$

It has been found simpler to change mixed numbers to improper fractions to multiply. For example, to multiply $1 \frac{2}{3} \times 4 \frac{1}{8}$, change the numbers to $5/3 \times 33/8$ then multiply as before.

To change a mixed number to an improper fraction, multiply the whole number by the denominator and add the result to the numerator.

To change $4 \frac{1}{8}$ to an improper fraction, multiply $8 \times 4 = 32$. Add $32 + 1 = 33$ and place the 33 over the denominator 8.

$$4 \frac{1}{8} = \frac{(8 \times 4) + 1}{8} = \frac{32 + 1}{8} = \frac{33}{8}$$

Division of common fractions

Division of fractions is accomplished by changing the problem to a multiplication problem. How this is done is shown in the following example.

Divide $\frac{3}{4}$ by $\frac{2}{3}$:

$$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1 \frac{1}{8}$$

The rule for division of fractions is: Write the problem as a division problem, then rewrite the problem as a multiplication problem by changing the sign to multiply and inverting the divisor. Then proceed as in multiplication.

If we write the above problem in a different form, we can show why the rule works.

$$\frac{\frac{3}{4}}{\frac{2}{3}} = \frac{\frac{3}{4} \times \frac{3}{2}}{\frac{2}{3} \times \frac{3}{2}} = \frac{\frac{3}{4} \times \frac{3}{2}}{1} = \frac{3}{4} \times \frac{3}{2}$$

The denominator $\frac{2}{3}$ was multiplied by a number that gave a product of 1.

$\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$. The numerator was also multiplied by the same number.

Therefore, the complex fraction $\frac{\frac{3}{4}}{\frac{2}{3}}$ was multiplied by 1 in the form.

$\frac{3}{2}$ and the value was not changed, only the form.

DECIMAL EQUIVALENTS

Decimal fractions and common fractions have the same meaning. They both represent numbers having a value less than one whole. If a group of students

were told to write the number three-tenths, some would write ' $\frac{3}{10}$ ', while others would write 0.3. Both would be correct.

It is important that students of industrial arts have a thorough understanding of both decimal and common fractions.

Many measuring instruments such as micrometers, vernier calipers and steel rules are graduated in decimal divisions. Others, such as rulers, steel squares, and tapes are graduated in common fractions.

The size of some materials used in industrial arts work, such as sheet metal and wire are in decimal fractions while angle iron, wood, and bolts are in common fractions.

Adjustment scales on woodworking machinery are graduated in common fractions whereas the scales on metalworking machinery are graduated in decimal fractions.

Drill bits come in either decimal or common fraction sizes.

It is clear from the above examples that work in industrial arts will include mathematical problems dealing with both decimal and common fractions and in many cases, with both in the same problem. Because of this, one must be able to transform a fraction from one form to the other.

Changing decimal fractions to common fractions

To change a decimal fraction to a common fraction, write the number as the numerator and the name as the denominator and reduce to lowest terms. The decimal fraction 0.25 is read twenty-five hundredths. Twenty-five is the number, hundredths is the name. In common fraction form, we would have $\frac{25}{100}$. Reduced to lowest

terms, the fraction would be $1/4$.

$$0.125 = \text{one hundred twenty five thousandths} = 125/1000 = 1/4$$

$$0.50 = \text{fifty hundredths} = 50/100 = 1/2$$

$$0.0625 = \text{six hundred twenty five ten thousandths} = 625/10000 = 1/16$$

Changing common fractions to decimal fractions

Referring to the previous illustration of place-value of decimal numbers, it will be noted that a decimal point divides the whole number part and the decimal part of a number. This indicates a decimal point is understood to be at the right-hand end of a whole number, so if it is needed, it can be so placed.

To change a common fraction to a decimal fraction, the whole number in the numerator must be divided into a number of equal parts as indicated by the number in the denominator. The rule then would be to change a common fraction to a decimal fraction, divide the numerator by the denominator.

Change $3/4$ to a decimal fraction.

$$\begin{array}{r} .75 \\ 4 \overline{) 3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$$3/4 = .75$$

Notice, a decimal point was placed after the 3 and enough zeros were added so the division by 4 could be continued until a remainder of zero was obtained. The addition of zeros after the decimal point does not change the value.

To change $1/16$ to a decimal, we must add 4 zeros.

$$\begin{array}{r}
 .0625 \\
 16 \overline{) 1.0000} \\
 \underline{96} \\
 40 \\
 \underline{32} \\
 80 \\
 \underline{80} \\
 0
 \end{array}$$

The measuring instruments which are graduated in common fractional parts use graduations of $1/2$, $1/4$, $1/8$, $1/16$, $1/32$, or $1/64$. These units are so common and the need to change them to decimal numbers so frequent that most shops and textbooks display a table showing their decimal values. This table is called Decimal Equivalents Table.

The decimal equivalent table can save many hours of computation and measuring, so the table is presented on the next page.

DECIMAL EQUIVALENTS TABLE

1/64	-----	.015625	33/64	-----	.515625
1/32	-----	.03125	17/32	-----	.53125
3/64	-----	.046875	35/64	-----	.546875
1/16	-----	.0625	9/16	-----	.5625
5/64	-----	.078125	37/64	-----	.578125
3/32	-----	.09375	19/32	-----	.59375
7/64	-----	.109375	39/64	-----	.609375
1/8	-----	.125	5/8	-----	.625
9/64	-----	.140625	41/64	-----	.640625
5/32	-----	.15625	21/32	-----	.65625
11/64	-----	.171875	43/64	-----	.671875
3/16	-----	.1875	11/16	-----	.6875
13/64	-----	.203125	45/64	-----	.703125
7/32	-----	.21875	23/32	-----	.71875
15/64	-----	.234375	47/64	-----	.734375
1/4	-----	.250	3/4	-----	.750
17/64	-----	.265625	49/64	-----	.765625
9/32	-----	.28125	25/32	-----	.78125
19/64	-----	.296875	51/64	-----	.796875
5/16	-----	.3125	13/16	-----	.8125
21/64	-----	.328125	53/64	-----	.838125
11/32	-----	.34375	27/32	-----	.84375
23/64	-----	.359375	55/64	-----	.859375
3/8	-----	.375	7/8	-----	.875
25/64	-----	.390625	57/64	-----	.890625
13/32	-----	.40625	29/32	-----	.90625
27/64	-----	.421875	59/64	-----	.921875
7/16	-----	.4375	15/16	-----	.9375
29/64	-----	.453125	61/64	-----	.953125
15/32	-----	.46875	31/32	-----	.96875
31/64	-----	.48438	63/64	-----	.984375
1/2	-----	.500	1	-----	1.000

ROUNDING DECIMAL NUMBERS

Sometimes we may not want as many decimal places in the decimal fraction as it takes to make the division come out even. In that case, we can round off the number to as many places as we need.

To round a decimal number:

1. Decide on the number of places needed.
2. If the next digit to the right is 5 or more, add one. If the digit is less than 5, do not add 1.
3. Then drop all remaining digits.

Round .0625 to the nearest hundredth.

Hundredths means we need two decimal places. The next digit is 2 which is less than 5 so we do not add 1; the desired number is .06.

Round .0625 to the nearest thousandths.

Thousandths means we need 3 decimal places. The next digit to the right is 5 so we add 1 and the desired number is .063.

.375 rounded to the nearest tenth is .4.

.01743 rounded to the nearest hundredth is .02.

.0997 rounded to the nearest thousandth is .100.

Rounding of decimals is used when the number of places is limited by the accuracy of measuring instruments or other limitations.

If a measuring instrument can only measure in thousandth, it would be a waste of time to make calculations with six or eight decimal places. In this case, all numbers should be rounded to the nearest thousandth.

If cost is being figured, it would be foolish to use more than two decimal places or hundredth because hundredths or cents is the smallest coin we use, so numbers representing money are always rounded to two decimal places.

FORMULAS

Plywood is sold by the square foot. If the cost of plywood is 16¢ per square foot and you wanted to buy a piece 2 feet by 4 feet, it would cost 8 square feet times 16¢ or $8 \times \$.16$, which would be \$1.28.

Each time you figure the cost of a different size piece of plywood or the cost of the same size piece of different kinds of plywood, you make the same problem set up. The difference in the problems is only that the numbers change. In other words, the number of square feet varies with the size of the piece and the price varies with the kind of plywood, but the problem is always worked the same way.

When it is found that a series of mathematical problems is set up the same way, the set up can be stated as a rule. For example, the cost of plywood is equal to the number of square feet of plywood times the price per square foot.

If we let 'P' stand for the price per square foot of plywood, 'N' stand for the number of square feet of plywood and 'C' stand for the cost of each piece of plywood, we could write the above rule as $N \times P = C$.

$N \times P = C$ is called a formula. It is a pattern to follow when figuring the cost of a given amount of plywood. The letters N, P and C are called variables because they vary depending on the particular problem.

There are many formulas used in the mathematical problems in industrial arts work so it is very important to understand formulas and know how to use them.

It is necessary that units of measure be known for each variable when using a formula. The importance of this can be shown by the formulas for figuring the number of board feet in a board.

$$B.F. = \frac{T \times W \times L}{144}$$

Where B.F. = number of board feet

T. = thickness in inches

W. = width in inches

L. = length in inches

$$B.F. = \frac{T \times W \times L}{144}$$

Where B.F. = number of board feet

T. = thickness in inches

W. = width in inches

L. = length in feet

HOW TO USE A FORMULA

The letters used in formulas are called literal numbers. They are replaced with actual numbers when the formula is applied to a specific problem.

Remember, a formula is a pattern showing how a particular problem should be set up. For instance, $A = \pi r^2$ is the formula for finding the area 'A' of any circle. Notice, there is no sign of operation between the π and r^2 . The absence of a sign of operation means to multiply. $A = \pi \times r^2$.

When literal numbers are used, the symbol for multiply could be taken as the letter x, so other means of indicating multiplication must be used. Multiplication is indicated by using no symbol, by using a dot between numbers and by using parenthesis.

$$A = lw \quad - \text{no symbol}$$

$$V = l.w.h \quad - \text{dot}$$

$$P = 2(l + w) \quad - \text{parenthesis}$$

Care must be exercised when using the above methods of indicating multiplication.

The 'no symbol' method cannot be used with numbers alone, as the numbers would appear as one number. Two times three would appear as 23.

The 'no symbol' method is used only with literal numbers.

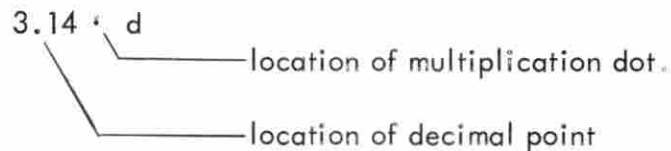
xy means x times y

$3b$ means 3 times b .

$4bc^2$ means 4 times b times c times c .

The dot is seldom used, but when it is used, it should always be placed in position half-way between top and bottom of the figures. Otherwise, it might be mistaken for a decimal point.

3.14 · d



location of multiplication dot.

location of decimal point

When parenthesis are used, the computation indicated inside the parenthesis must be performed first and then multiply by the number outside the parenthesis.

$$2(4 - 1) = 2(3) = 6$$

When using a formula, a systematic procedure should be used to avoid making mistakes.

1. Write the formula.
2. List the values for the known variables or literal numbers in the formula.
3. Write the formula again, replacing the variables with given numbers.
4. Perform the required operations.

5. Write the answer labeled with the correct units of measure.

Using the formula $A = \pi r^2$, find the area of the circle having a radius of 13 inches.

Step 1	$A = \pi r^2$
Step 2	$\pi = 3.14$
	$r = 13 \text{ inches}$
Step 3	$A = 3.14 \times 13 \times 13$
Step 4	$A = 3.14 \times 169 = 530.66$
Step 5	$A = 530.66 \text{ square inches}$

Find the volume of a rectangular container 16 inches long, 5 inches wide and 3 inches high. $V = lwh$; where l = length; w = width; h = height; V = volume.

Step 1	$V = lwh$
Step 2	$l = 16 \text{ inches}$
	$w = 5 \text{ inches}$
	$h = 3 \text{ inches}$
Step 3	$V = 16 \times 5 \times 3$
Step 4	$V = 16 \times 15 = 240$
Step 5	$V = 240 \text{ cubic inches}$

CHANGING A FORMULA

A formula can be used to find the value of any of its variables if the value of all other variables is known. For example, if the area and length of a rectangle is known, we can use the formula $A = lw$ to find the value of the width, but first,

the formula must be changed. Changes in formulas can be made by making use of some axioms of equations.

An equation is two mathematical expressions which are equal. We have an equation whenever the symbol $=$ is used. Therefore, all formulas are equations.

The axioms of equations which will be needed to change formulas are:

1. Addition axioms -- The same quantity can be added to both sides of an equation.
2. Subtraction axiom -- The same quantity can be subtracted from both sides of an equation.
3. Multiplication axiom -- Both sides of an equation can be multiplied by the same quantity.
4. Division axiom -- Both sides of an equation can be divided by the same quantity.

Find the length of a rectangle if the width is 6 inches and the area is 24 square inches.

$A = lw$ is the formula to find the area of a rectangle. If we use the division axiom and divide both sides of the formula by w , we will have a formula for finding the length.

$$A/w = l\cancel{w}/\cancel{w} \text{ or } l = A/w.$$

Now, solve the problem

$$l = A/w$$

$$A = 24 \text{ square inches}$$

$$w = 6 \text{ inches}$$

$$l = 24/6 = 4$$

$$l = 4 \text{ inches.}$$

Construct a wood box 12 inches high and 12 inches wide that will hold exactly one bushel of apples.

The problem is to determine how long to make the box. The box is rectangular in shape and bushel is to measure volume, so we will use the formula $V = lwh$ where V = volume; l = length; w = width and h = height.

Change the formula to find l .

$$V = lwh$$

$$V/wh = lwh/wh$$

A table of weights and measures shows that 1 bushel = 2150 cubic inches.

Now we have all the information to solve the problem.

$$l = V/wh$$

$$l = 2150/12 \times 12$$

$$l = 2150/12 \times 12 = 2150/144 = 14.931$$

$$l = 14.931 \text{ inches}$$

If we wish to change the decimal number 14.931 inches to a fraction, we can refer to the decimal equivalent chart and find the nearest decimal number to .931. The nearest number is .9375 which corresponds to $15/16$. The length of the box would then be $14 \frac{15}{16}$ inches.

GEOMETRIC CONSTRUCTION

There are many geometric constructions which are useful in industrial arts work. They are useful for making designed, drawing plans, making layouts on

material and checking work.

Most geometric constructions can be made with the use of only a pencil, compass and a straight edge or ruler.

No attempt will be made to include all geometric constructions in this text. Only the most common and most useful will be included.

How to copy a line segment

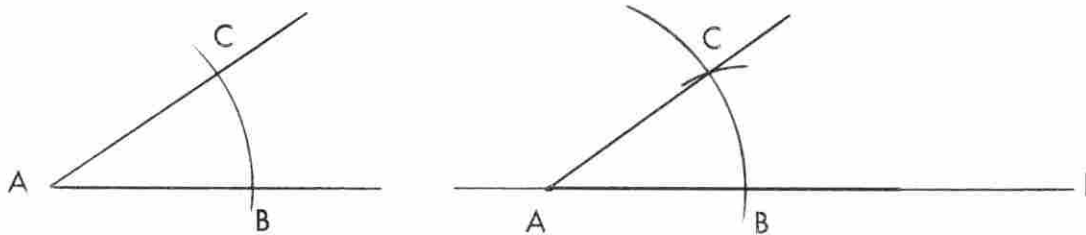
Given line segment AB



1. Draw a line l longer than AB .
2. Select a point A on the left line of line l and label it A .
3. Set one point of the compass on point A of line segment AB and adjust compass so other point is at point B .
4. Place point of compass at point A on line l and strike an arc intersecting line l with the pencil end of compass.
5. Label the point of intersection B .
6. The line segment AB on line l is a copy of the given line segment.

How to copy an angle

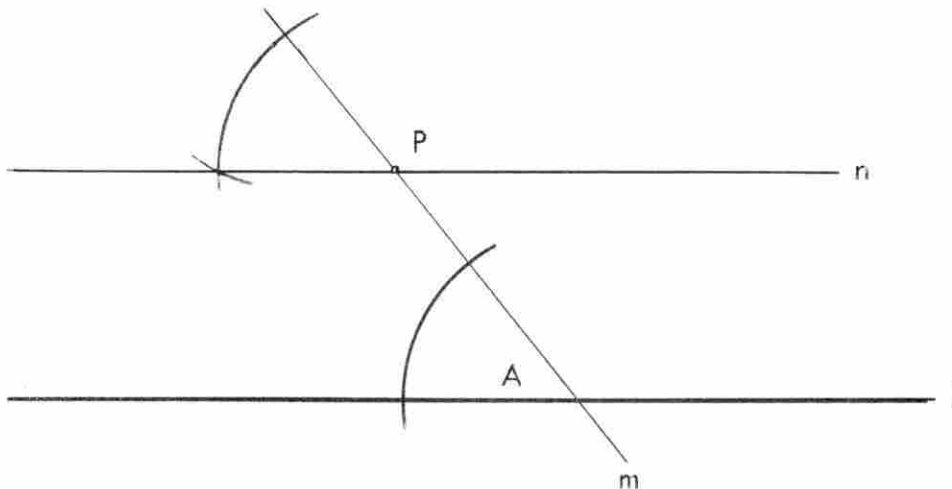
Given angle A



1. Draw line l .
2. Select a point on line l and label point A .
3. Set compass to a convenient radius and with point of compass at A draw an arc intersecting the sides of the given angle at B and C .
4. With point of compass at point A on line l draw the same arc intersecting line l at B .
5. On the given angle set the radius of the compass to the distance from point B to C .
6. With compass point at point B on line l , draw an arc intersecting the arc through B . Mark point of intersection of arc point C .
7. Draw a straight line from point A through point C which completes the angle.

How to construct a line parallel to a given line through a point not on the line

Given line l and point P

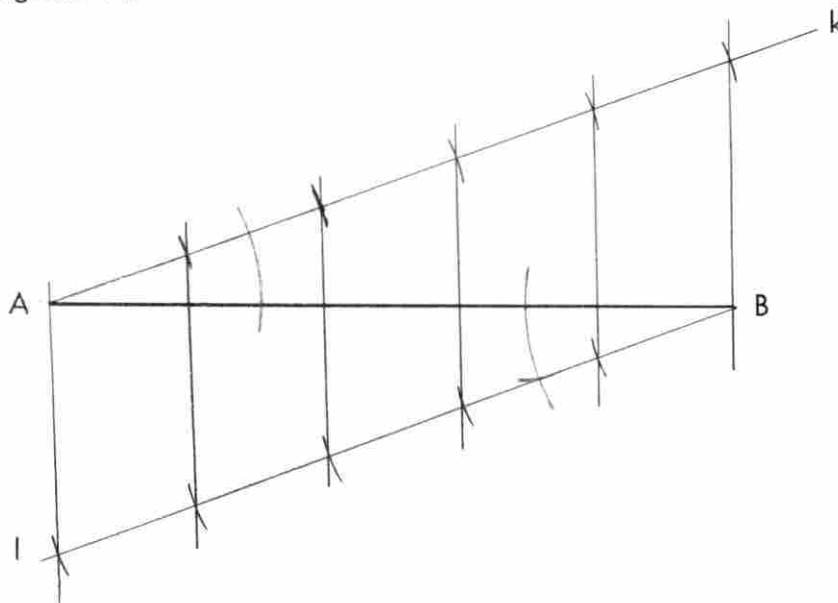


1. Draw a line m passing through point P and line l at any convenient angle A .

2. Using point P as the vertex, copy angle A. Use procedure for copying an angle.
3. The line n, which forms the angle at point P is parallel to line l.

How to divide a line segment into equal parts

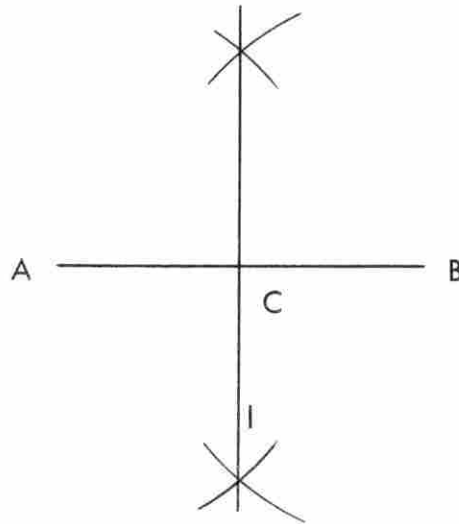
Given line segment AB



1. Draw line l, from end A of line segment AB, at any convenient angle.
2. Copy angle A at point B as shown, forming line k.
3. Using a compass and starting at points A and B, mark the desired number of equal parts on line k and l.
4. Draw lines as shown, connecting marks on line k with marks on line l.
5. The intersections of these lines with line segment AB are points dividing AB into the desired number of equal parts.

How to bisect a line segment

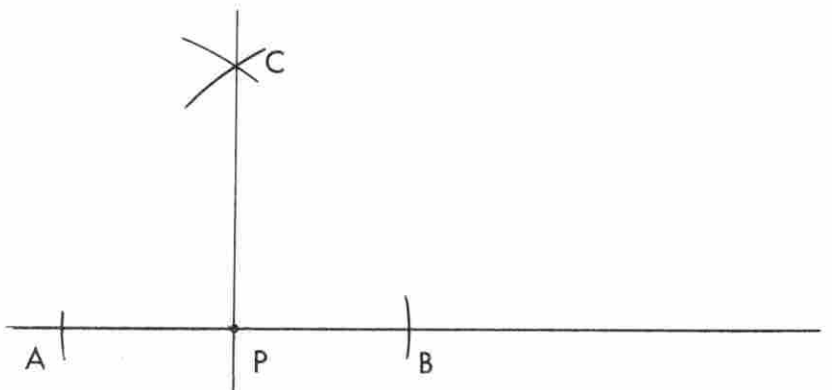
Given line segment AB



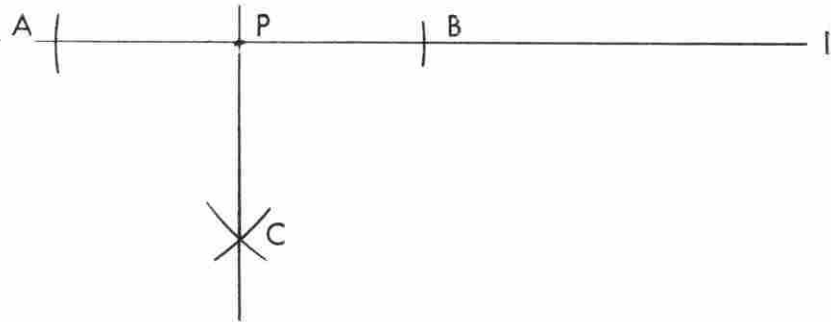
1. Set radius of the compass larger than the length of AB.
2. With point of the compass at A, draw an arc above and below the center of line segment AB.
3. Using the same radius, place the point of the compass at B, draw arcs above and below line segment AB intersecting the first arcs.
4. Draw line l through the intersections of the arcs.
5. Point C, where l intersects AB, is the bisector of line segment AB.

How to construct a perpendicular to a line through a point on the line

Given line l and point P.



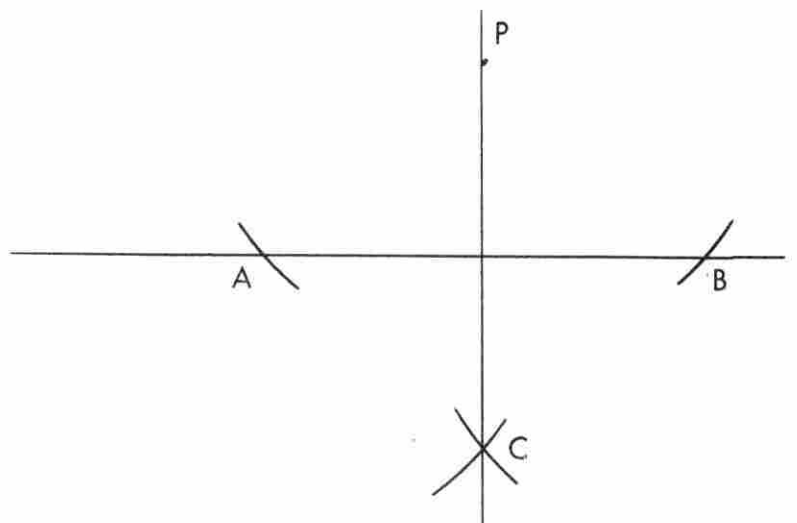
Alternate method:



1. Using the same radius and point P as center, draw arcs intersecting line l at A and B.
2. Using A and B as centers and a radius greater than length AP , draw arcs intersecting at point C.
3. Draw a line through points C and P.
4. This line passes through point P and is perpendicular to line l.

How to construct a perpendicular to a line through a point outside the line

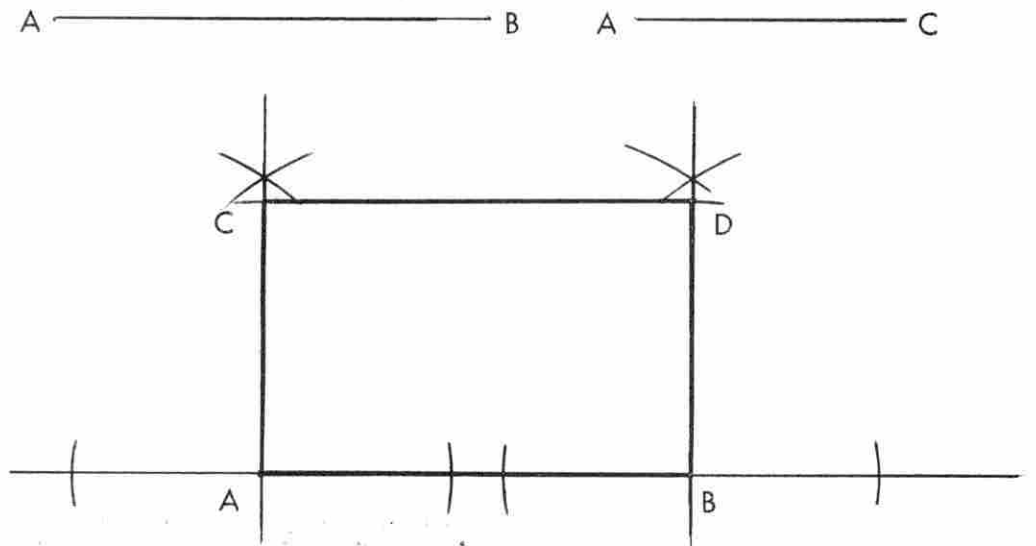
Given line l and point P.



1. With a radius greater than the distance from point P to line l and using point P as center, draw an arc intersecting line l at A and B.
2. Using any convenient radius and points A and B as centers draw arcs with the same radius intersecting at point C.
3. Draw a line passing through points P and C.
4. This line passes through point P and is perpendicular to line l.

How to construct a rectangle

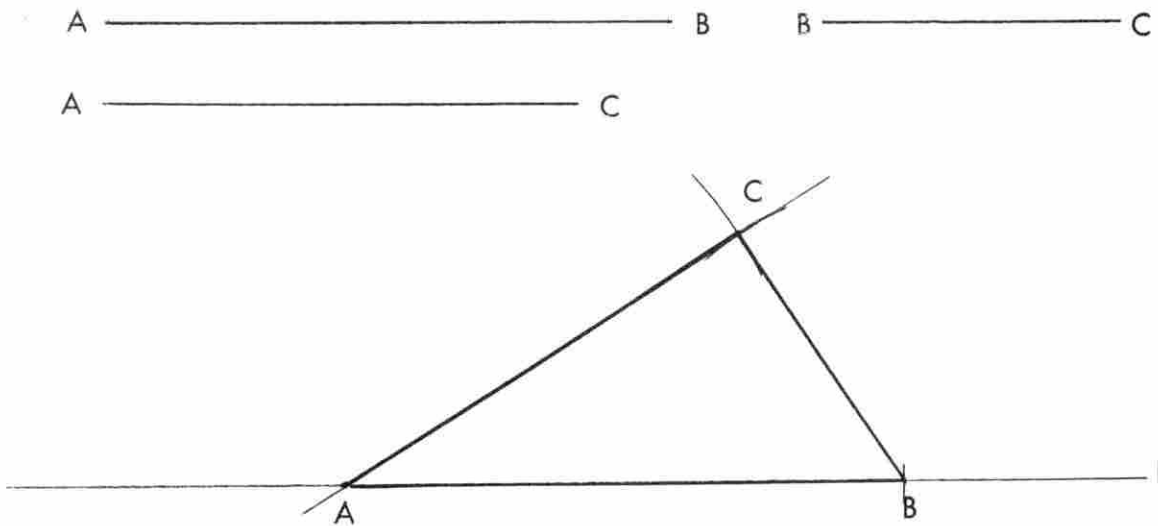
Given length AB and width AC



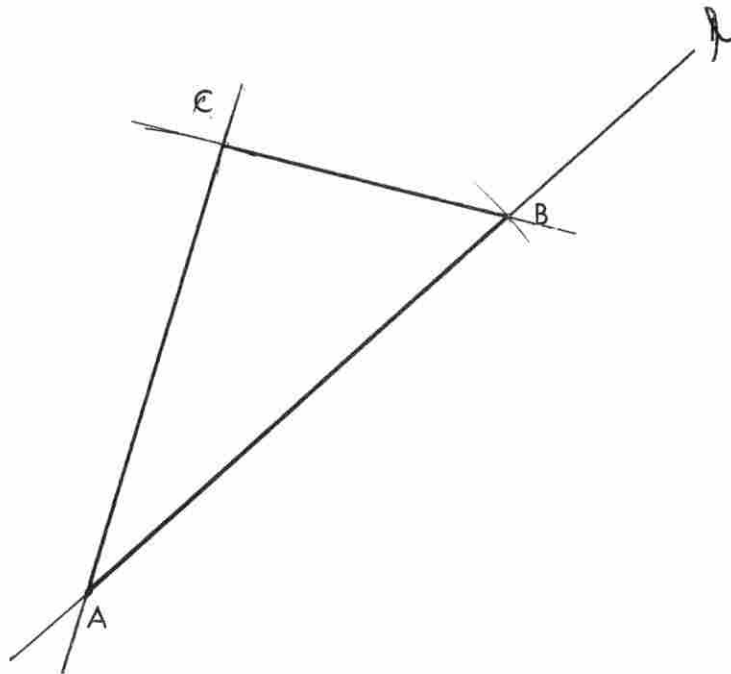
1. Draw line l and mark length AB.
2. Construct a perpendicular line l at points A and B.
3. Using A and B as center and length AC as the radius, draw arc intersecting the perpendicular lines at C and D.
4. Draw a line from C to D.

How to construct a triangle if three sides are known

Given sides of triangle, AB, AC, and BC.



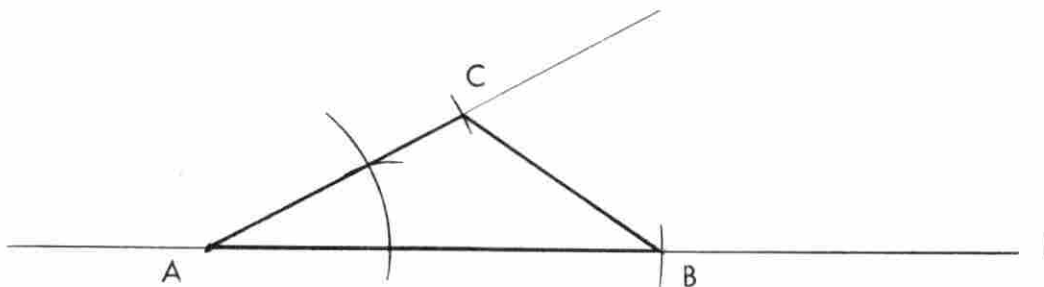
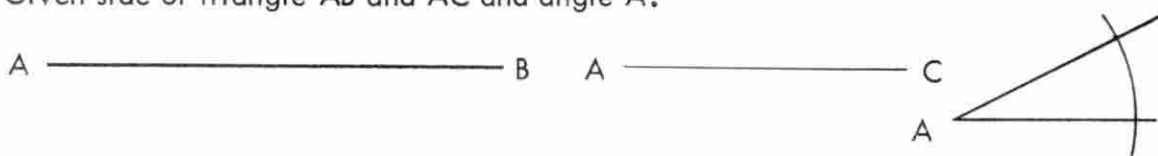
Students usually develop a habit of starting geometric constructions with a horizontal line as was done in the construction above. Later, we will have to perform some construction beginning in different positions. It would, therefore, be advisable to practice. Below is the same construction in a different position.



1. Draw line l and copy side AB on line l .
2. With compass set to a radius equal to length AC and center at point A on line l , make an arc as shown.
3. With compass set to a radius equal to length BC and center a point B on line l , make an arc intersecting the first arc at point C .
4. Draw straight lines between points A and C and between B and C , completing the triangle.

How to construct a triangle if two sides and the included angle are known

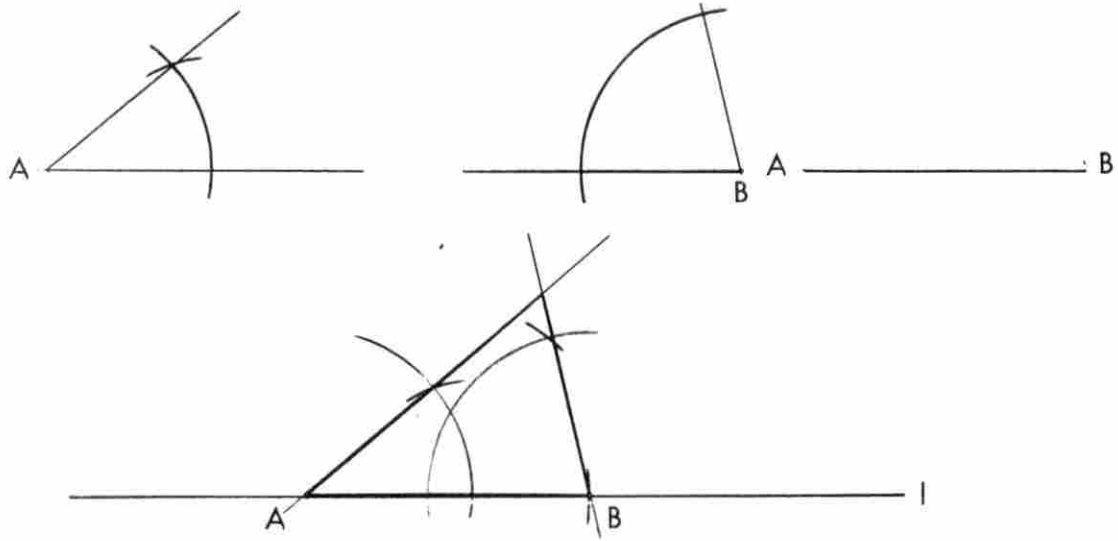
Given side of triangle AB and AC and angle A .



1. Draw line l and copy side AB on line l .
2. Copy angle A using point A on line l as vertex and using line l as one side of the angle.
3. Copy side AC on line forming side of angle A and mark point C .
4. Connect points B and C with a straight line completing the triangle.

How to construct a triangle if two angles and the included side are known

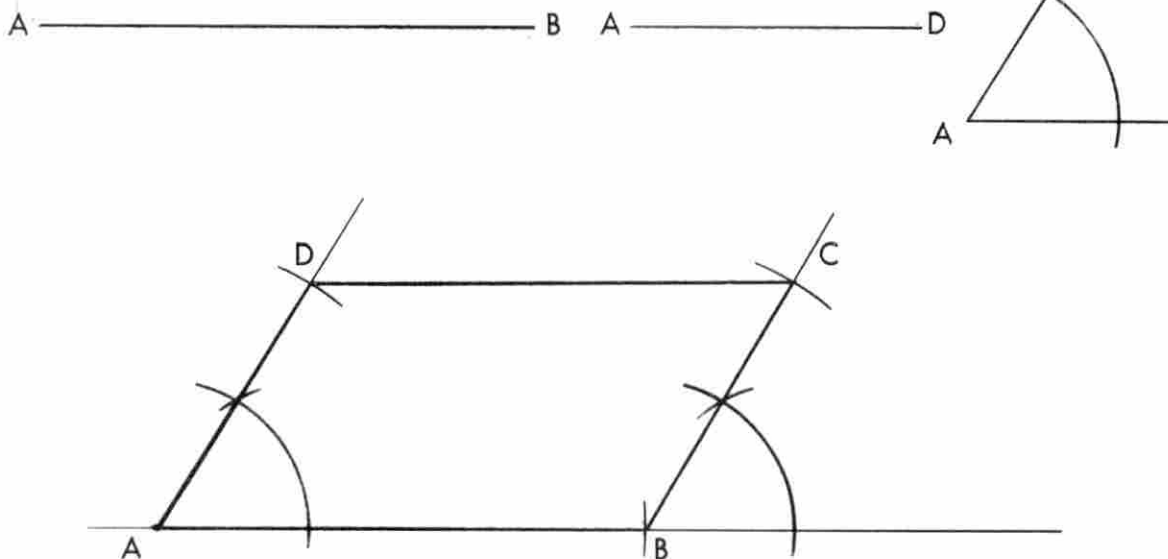
Given angles A and B and side AB



1. Draw line l and copy side AB on line l .
2. Copy angle A on line l using point A as the vertex.
3. Copy angle B on line l using point B as the vertex.
4. Extend the sides of angles A and B until they intersect at point C completing the triangle.

How to construct a parallelogram

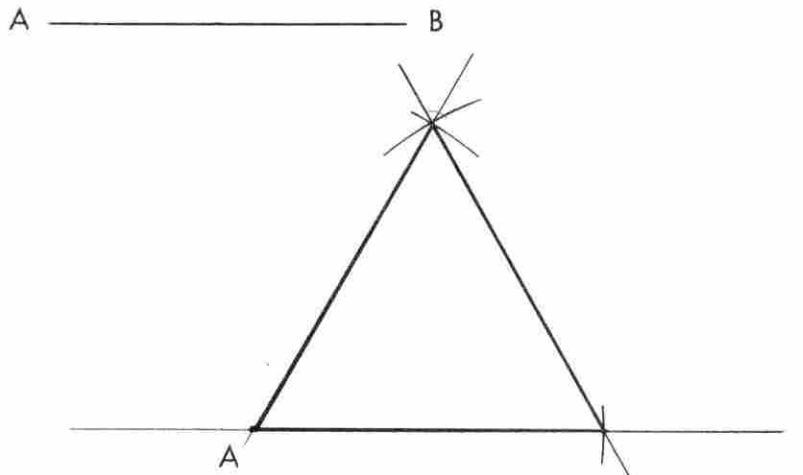
Given side AB , AD and angle A



1. Draw line l and copy side AB on l .
2. Using line l as one side, copy angle A at points A and B on line l as shown.
3. Copy side AD on the sides of the two angles using points A and B as end points forming sides AD and BC .
4. Connect points D and C with a straight line completing the parallelogram.

How to construct an equilateral triangle

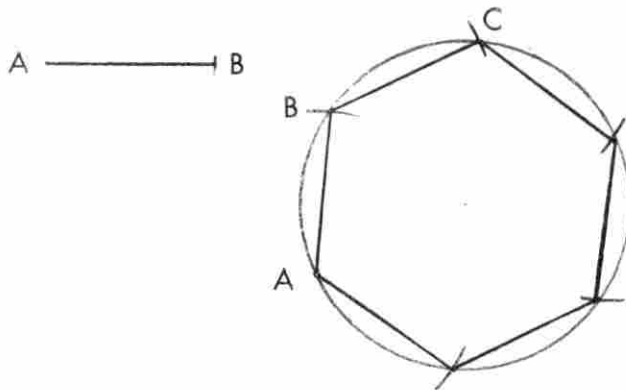
Given side AB of triangle



1. Draw line l and copy length of side AB on line l .
2. With compass set to a radius equal to the length of side AB and using points A and B on line l as centers, draw arcs intersecting at point C .
3. Draw straight lines connecting points A and C and points B and C completing the equilateral triangle.

How to construct a hexagon

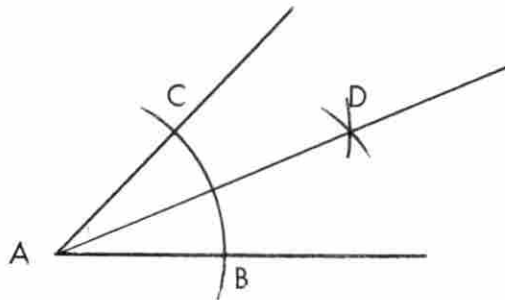
Given side AB of hexagon



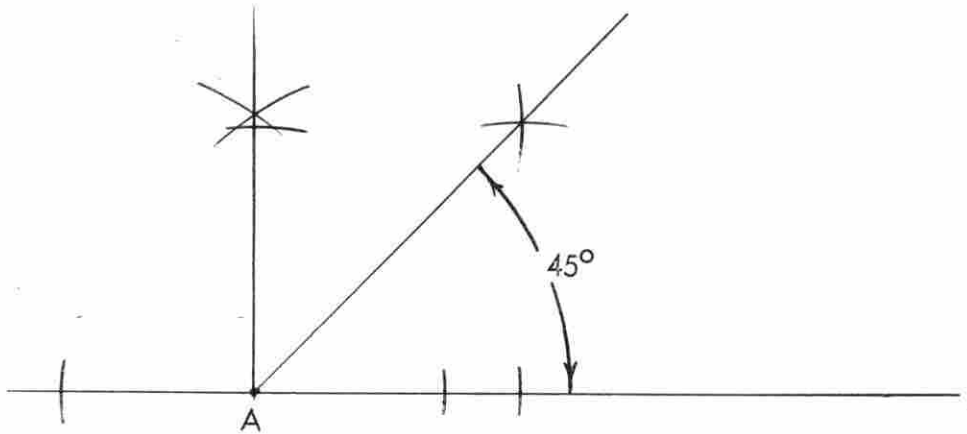
1. Set radius of compass to length of side AB and draw a circle.
2. Select and mark point A on the circle.
3. With the same radius as used to draw the circle and with point A as center make an arc intersecting the circle at point B.
4. Using point B as center make an arc intersecting the circle, at point C.
5. Continue step 4 until the circle has been divided into six equal parts.
6. Connect consecutive points with straight lines forming a hexagon.

How to bisect an angle

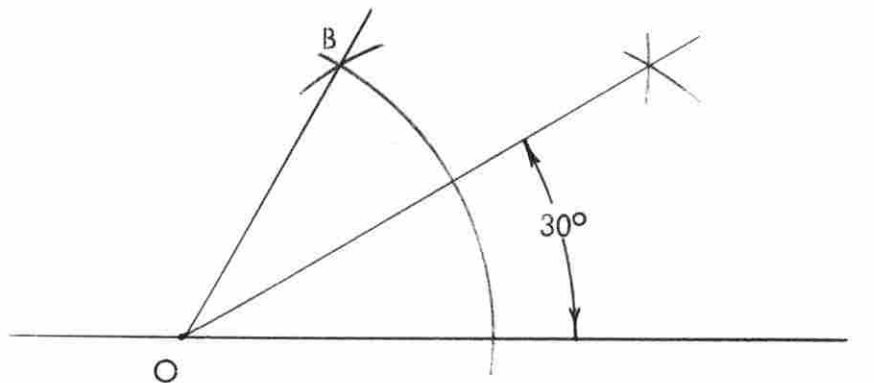
Given angle A



1. Using point A as center and any convenient radius, draw an arc intersecting the sides of the angle at points B and C.
2. With a radius larger than length AB and using points B and C as centers, draw arcs intersecting at point D.
3. Draw a line through points A and D which will bisect the angle.

How to draw a 45° angle

1. Draw line l and select a point A .
2. Construct a perpendicular to line l , passing through point A .
3. Bisect one of the angles.

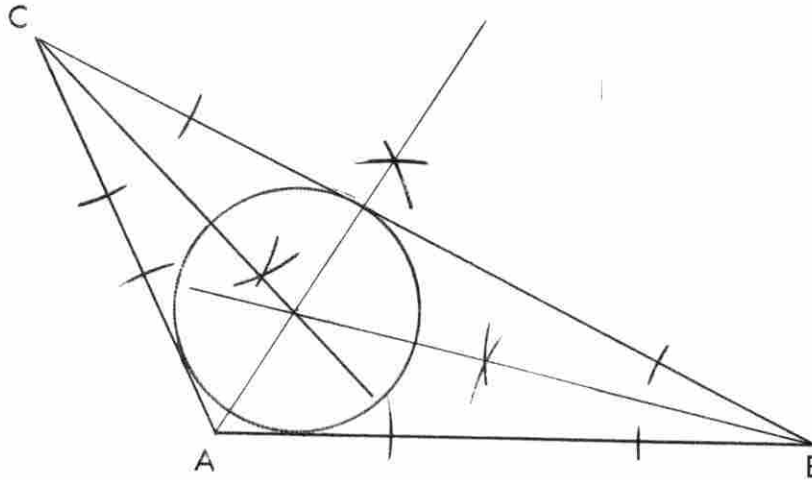
How to construct a 30° angle

1. Draw line l and select point O .
2. Using any convenient radius and point O as center, draw an arc intersecting line l at point A .
3. Using the same radius and point A as center, draw an arc intersecting the first arc at point B .
4. Draw a line through points O and B making angle AOB

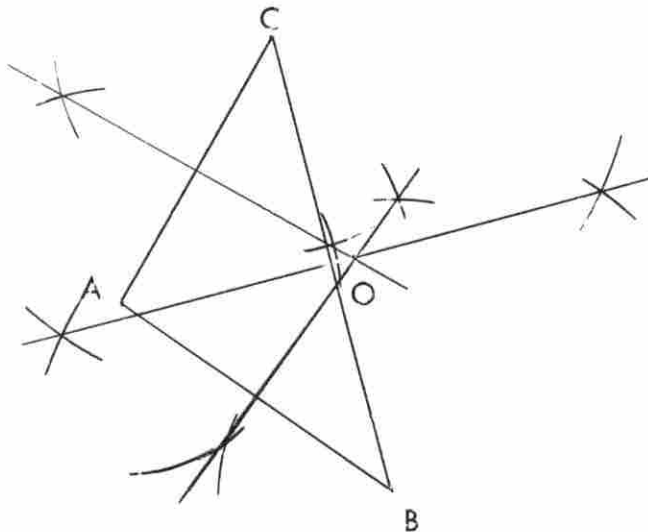
5. Bisect angle AOB

How to inscribe a circle in a triangle

Given triangle ABC



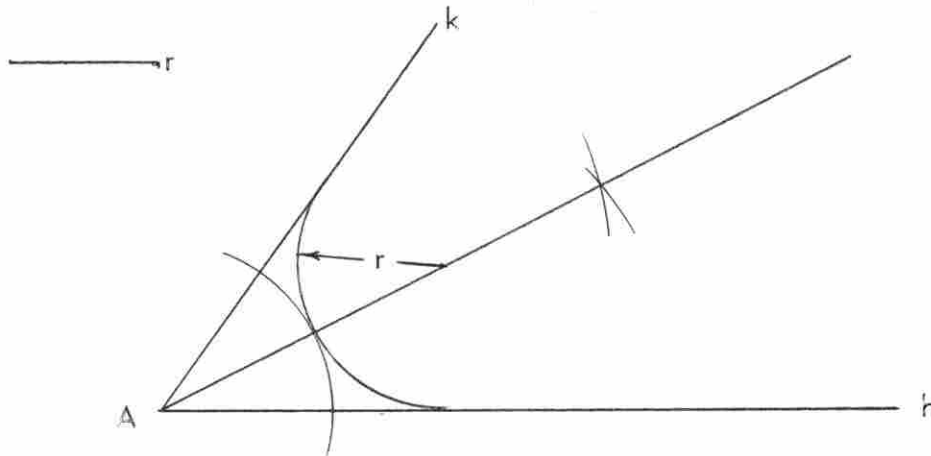
1. Bisect each angle of the triangle.
2. Extend the bisectors of each angle until all three intersect at the same point O.
3. Using a radius equal to the distance from point O to a side of the triangle and using point O as center, draw a circle.

How to circumscribe a circle about a triangle

1. Bisect each side of triangle ABC
2. Extend the bisectors until all three intersect at point O.
3. Using a radius equal to the distance from point O to any vertex and using point O as center, draw a circle.

How to blend an arc of given radius between intersecting lines

Given lines h and l intersecting at point A and radius r.



1. Bisect the angle formed by lines h and k.
2. Using radius r, locate a center on the bisector such that the distance from the center to sides h and k will be equal to r, then draw the arc.

TRIGONOMETRY

Many design and construction problems of industrial arts work can be solved by the use of three basic trigonometric formulas. These are the sine, cosine and tangent formulas.

The names of these formulas are commonly abbreviated to sin, cos and tan respectively.

The use of the sin, cos and tan formulas, as discussed here, only apply to right triangles.

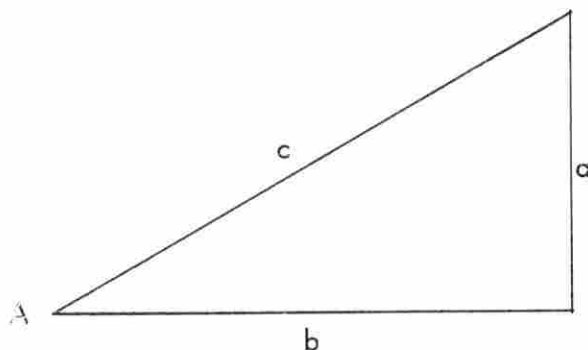
The symbol \angle will be used to mean angle.

Let A = either acute angle of a right triangle.

a = side opposite $\angle A$.

b = side adjacent to $\angle A$.

c = hypotenuse of the right triangle.



Sine formula:

$$\sin \angle A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

Cosine formula

$$\cos \angle A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

Tangent formula

$$\tan \angle A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$

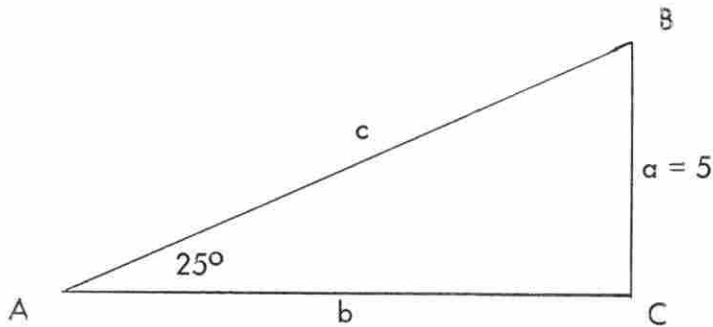
The ratio a/c is called the sine function; b/c is called the cosine function and a/b is called the tangent function. Collectively, they are called trigonometric functions.

The values of the trigonometric functions are always the same regardless

of the lengths of the sides. Therefore, the value of each function can be computed and listed in a table for future references. The computations have been made for angles from 1° to 90° and are listed in the table on the next page.

Every right triangle has six parts, three sides and three angles. If any two parts of a right triangle are known, all other parts can be found by using the three trigonometric formulas.

Find the value of all parts of triangle ABC.



To find C, use the sine formula.

$$\sin \angle A = a/c$$

$$\sin 25^\circ = 5/c \quad (\text{from table } \sin 25^\circ = .4226)$$

$$.4226 = 5/c$$

$$c = 5/.4226$$

$$c = 11.8$$

TRIGONOMETRIC FUNCTIONS

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	46°	.7193	.6947	1.0355
2°	.0349	.9994	.0349	47°	.7314	.6820	1.0724
3°	.0523	.9986	.0524	48°	.7431	.6891	1.1106
4°	.0698	.9976	.0699	49°	.7547	.6561	1.1504
5°	.0872	.9962	.0875	50°	.7660	.6428	1.1918
6°	.1045	.9945	.1051	51°	.7771	.6293	1.2349
7°	.1219	.9925	.1228	52°	.7880	.6157	1.2799
8°	.1392	.9903	.1405	53°	.7986	.6018	1.3270
9°	.1564	.9877	.1584	54°	.8090	.5878	1.3764
10°	.1736	.9848	.1763	55°	.8192	.5736	1.4281
11°	.1908	.9816	.1944	56°	.8290	.5592	1.4826
12°	.2079	.9781	.2126	57°	.8387	.5446	1.5399
13°	.2250	.9744	.2309	58°	.8480	.5299	1.6003
14°	.2419	.9703	.2493	59°	.8572	.5150	1.6643
15°	.2588	.9659	.2679	60°	.8660	.5000	1.7321
16°	.2756	.9613	.2867	61°	.8746	.4848	1.8040
17°	.2924	.9563	.3057	62°	.8829	.4695	1.8807
18°	.3090	.9511	.3249	63°	.8910	.4540	1.9626
19°	.3256	.9455	.3443	64°	.8988	.4384	2.0503
20°	.3420	.9397	.3640	65°	.9063	.4226	2.1445
21°	.3584	.9336	.3839	66°	.9135	.4067	2.2460
22°	.3746	.9272	.4040	67°	.9205	.3907	2.3559
23°	.3907	.9205	.4245	68°	.9272	.3746	2.4751
24°	.4067	.9135	.4452	69°	.9336	.3584	2.6051
25°	.4286	.9063	.4663	70°	.9397	.3420	2.7475
26°	.4384	.8988	.4877	71°	.9455	.3256	2.9042
27°	.4540	.8910	.5095	72°	.9511	.3090	3.0777
28°	.4695	.8829	.5317	73°	.9563	.2924	3.2709
29°	.4848	.8746	.5543	74°	.9613	.2756	3.4874
30°	.5000	.8660	.5774	75°	.9659	.2588	3.7321
31°	.5150	.8572	.6009	76°	.9703	.2419	4.0108
32°	.5299	.8480	.6249	77°	.9744	.2250	4.3315
33°	.5446	.8387	.6494	78°	.9781	.2079	4.7046
34°	.5592	.8290	.6745	79°	.9816	.1908	5.1446
35°	.5736	.8192	.7002	80°	.9848	.1736	5.6713
36°	.5878	.8090	.7265	81°	.9877	.1564	6.3138
37°	.6018	.7986	.7536	82°	.9903	.1392	7.1154
38°	.6157	.7880	.7813	83°	.9925	.1219	8.1443
39°	.6293	.7771	.8098	84°	.9945	.1045	9.5144
40°	.6428	.7660	.8391	85°	.9962	.0872	11.4301
41°	.6561	.7547	.8693	86°	.9976	.0698	14.3007
42°	.6691	.7431	.9004	87°	.9986	.0523	19.0811
43°	.6820	.7314	.9325	88°	.9994	.0349	28.6363
44°	.6947	.7193	.9657	89°	.9998	.0175	57.2900
45°	.7071	.7071	1.0000	90°	1.0000	.0000	

To find b , use the tangent formula.

$$\tan \angle A = 2/b$$

$$\tan 25^\circ = 5/b \quad (\text{from table } \tan 25^\circ = .4663)$$

$$.4663 = 5/b$$

$$b = 5/.4663$$

$$b = 10.5$$

To find $\angle B$ use the cosine formula.

$$\cos \angle B = \frac{\text{side adjacent to } \angle B}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos \angle B = 5/11.8$$

$$\cos \angle B = .4237$$

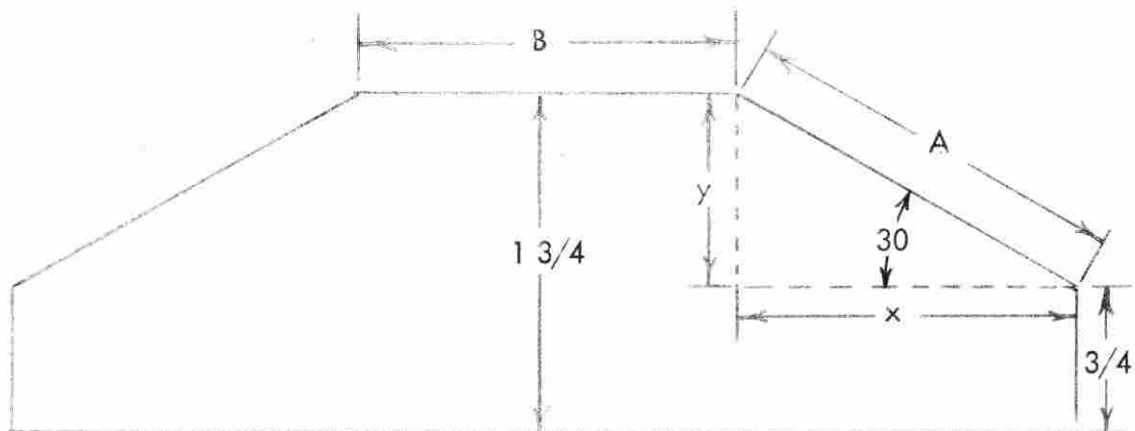
Find the number closest to .4237 in the cosine column of the table of trigonometric functions. It is found to be closest to .4226 which is the cosine of 65° . Angle B is then 65° .

The value of $\angle C$ is already known to be 90° because triangle ABC is a right triangle.

A practical problem involving the use of trigonometric functions can be illustrated by considering the drawing below. The drawing is of the end piece of the lid of a tool box.

The $3/4"$, $1\ 3/4"$, $5\ 1/2"$ and 30° dimensions are given, but dimension A and B must be calculated in order to lay out this pattern on sheet metal. Dimensions A and B are also needed to make a layout for the tool box top which will fit around these ends.

2 required



To calculate dimensions A and B, draw in the dotted line x parallel to the base line and the dotted line y perpendicular to x, thus completing a right triangle.

Dimension y is determined by subtracting $3/4"$ from $1\ 3/4"$.

$$y = 1\ 3/4 - 3/4 = 1".$$

Use the sine formula to determine the length of A.

$$\sin 30^\circ = y/A$$

$$.500 = 1/A$$

$$A = 1/.500 = 2"$$

Use the cosine formula to determine length of x.

$$\cos 30^\circ = x/A$$

$$.866 = x/2$$

$$x = 1.732" \text{ or from table of decimal equivalents,}$$

$$1\ 47/64".$$

Dimension B can be calculated by subtracting 2x from 5 1/2".

$$B = 5 \frac{1}{2} - 2x$$

$$B = 5 \frac{1}{2} - 2 (1 \frac{47}{64})$$

$$B = 5 \frac{1}{2} - 3 \frac{15}{32}$$

$$B = 2 \frac{1}{32}"$$

SUMMARY

The objectives of this study were:

1. To determine the need for a unit in shop mathematics for industrial arts classes based on:
 - a. Results of testing high school industrial arts students.
 - b. Opinions of industrial arts teachers.
 - c. Opinions of supervisors in industry.
2. To determine areas of weakness in mathematical ability of high school students by testing.
3. To determine areas of weakness in mathematical ability of semi-skilled employees in industry by survey of supervisors.
4. To develop a text on industrial arts mathematics based on the needs revealed by the above investigation.

A mathematics test was developed which would indicate the abilities of high school industrial arts students. The test was designed to reveal the weak areas of students' knowledge of mathematical operations.

The test results showed high school industrial arts students to be very low in mathematical ability. Students were able to perform the basic operations of addition, subtraction, multiplication and division of whole numbers, but their mathematical ability decreased sharply when decimal numbers and common fractions were involved. The ability to solve problems by the use of formulas proved to be almost nonexistent.

A survey of industrial arts teachers was made by questionnaire. The opinion of the teachers surveyed gave the same indication of student mathematical

abilities as the test. Teachers felt that greater interest to learn mathematics would be shown by students if it were taught as a unit in industrial arts courses and related to the activities of the course. The survey strongly indicated a desire for a text designed for such a unit.

It was decided to survey supervisors in industry as to their opinions on the subject of this study. A questionnaire was used to gather the required information.

According to supervisors surveyed, the semi-skilled employees in industry were low in mathematical ability, but not quite as low as industrial arts students. This slightly-higher rating for semi-skilled employees is probably due to selectivity in hiring personnel for industrial positions. However, supervisors felt that, in general, better mathematical preparation of students would be desirable. They also indicated a belief that a unit in mathematics in industrial arts courses would better prepare students for employment in industry.

Based on the results of this study, the text developed contains sections on the following:

1. Whole numbers
2. Decimal numbers
3. Common fractions
4. Decimal equivalents
5. Rounding numbers
6. Formulas
7. Basic algebra

The other sections were included in the text because of their usefulness

in design and layout work. These sections were on geometric construction and basic trigonometry.

LITERATURE CITED

1. Chrysler Corporation. Math problems from industry. Detroit, Michigan, Author. c1956.
2. Ferirer, John L. Industrial arts woodworking. 2nd ed. Peoria, Illinois, Chas. A. Bennett Co., Inc. c1965.
3. Feirer, John L. Workbook for industrial arts woodworking. Peoria, Illinois, Chas. A. Bennett Co., Inc. c1965.
4. New York State Vocational and Practical Arts Association. Simplified sheet metal mathematics. Albany, New York, Delmar Publishers, Inc. 1960.
5. Walker, John R. Modern metalworking materials, tools and procedures. Homewood, Illinois, The Goodheart-Willcox Company, Inc. c1965.

APPENDIX A

INDUSTRIAL ARTS MATHEMATICS TEST

Write the numbers stated in problems 1, 2, and 3.

1. Two thousand six hundred thirty two.
2. Fifty six and ninety five thousandths (decimal).
3. Seven and thirteen sixteenths (common fraction).

4. Add:

$$\begin{array}{r} 84.9 \\ 6.750 \\ \hline 147.98 \end{array}$$

5. Add:

$$\begin{array}{r} 732 \\ 6,829 \\ \hline 47 \end{array}$$

6. Add 163 and 1,729:

7. Add 2.37 to 28.518:

8. Subtract:

$$\begin{array}{r} 1,246 \\ - 984 \\ \hline \end{array}$$

9. Subtract:

$$\begin{array}{r} 29.145 \\ - 7.83 \\ \hline \end{array}$$

10. Subtract 479,871 from 1,241,299:

11. Subtract 27.26 from 681.5:

12. Multiply:

$$\begin{array}{r} 281 \\ \times 67 \\ \hline \end{array}$$

13. Multiply:

$$\begin{array}{r} 2.643 \\ \times 3.21 \\ \hline \end{array}$$

14. Multiply 6421 by 43:

15. Multiply 2.67 by 87.2:

16. Divide:

$$26 \overline{) 364}$$

17. Divide:

$$.4 \overline{) 1.28}$$

18. Divide 484 by 2141:

19. Divide 9.42 by 18.84:

20. Change $7/8$ to a decimal fraction:

21. $1/4 + 5/8 =$

22. $7/16 + 7/8 =$

23. $2 \frac{1}{16} + 14 \frac{7}{8} =$

24. $13/16 - 3/8 =$

25. $3 - 7/8 =$

26. $14 \frac{13}{16} - 9 \frac{3}{8} =$

27. $1/3 \times 3/4 =$

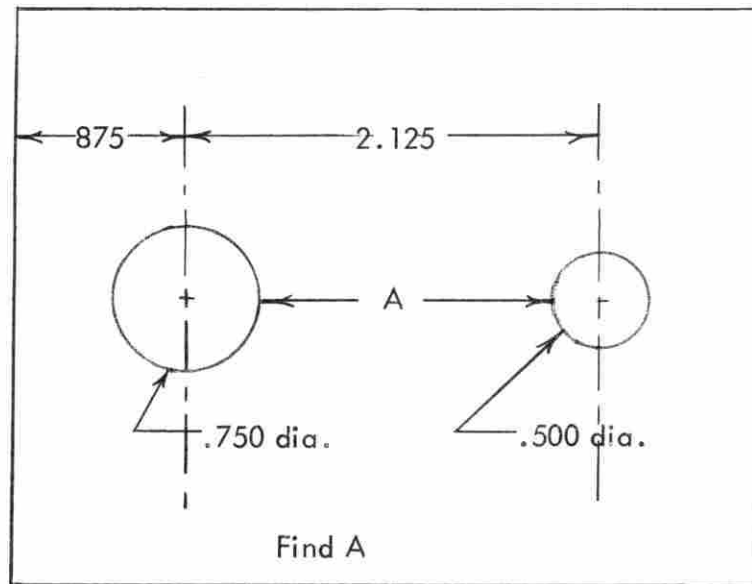
28. $7/4 \times 7/8 =$

29. $1 \frac{3}{16} \times 4 \frac{5}{8} =$

30. $3/8 \div 2 =$

31. $3\frac{7}{16} \div 1\frac{5}{8}$

32.



33. Using the formula $\frac{N \times T \times W \times L}{12} = \text{No. of board feet}$, when $N = \text{no. of pieces}$, $T = \text{thickness in inches}$, $W = \text{width in inches}$ and $L = \text{length in feet}$, find the no. of board feet in 8 pieces of wood $3/4" \times 2" \times 10'$.
34. $1/R_t = 1/R_1 = 1/R_2 = 1/R_3$ is the formula for finding total resistance in a parallel DC circuit. Find R_t if $R_1 = (100) \text{ ohms}$, $R_2 = (200) \text{ ohms}$ and $R_3 = (300) \text{ ohms}$.

APPENDIX B

QUESTIONNAIRE FOR INDUSTRIAL ARTS TEACHERS

1. Check the blank which most nearly represents the ability of your students to work mathematical problems related to industrial arts work.

Very low _____ Low _____ Average _____ Above average _____

2. Check the type of mathematics problems which are difficult for your students.

_____ Addition
 _____ Subtraction
 _____ Multiplication
 _____ Division
 _____ Fractions
 _____ Percentage
 _____ Using a formula
 _____ Algebraic operations
 _____ Others (please list) _____

3. Do you think your students would have more interest in learning the above operations if they were taught as a unit in industrial arts courses, using problems related to their work? yes _____ no _____

4. Would a short text containing instruction in the fundamental operations of mathematics and practical industrial arts mathematical problems be a useful aid in teaching mathematics to industrial arts students? yes _____ no _____

5. Do you feel there is a need for such a text? yes _____ no _____

6. If yes, check the items that should be included:

_____ Addition
 _____ Subtraction
 _____ Multiplication
 _____ Division
 _____ Fractions
 _____ Percentage
 _____ Using a formula
 _____ Algebraic operations
 _____ Others (please list) _____

APPENDIX C

SUPERVISOR'S QUESTIONNAIRE

1. How many employees do you supervise? _____
Are they skilled _____, semi-skilled _____, unskilled _____.
2. In what general area do these people work? (wood, metal, electronics)
_____.
3. Are these people required to work mathematics problems in connection with their work? _____
4. If no, is it because of low ability in mathematics? yes _____ no _____
5. If yes, do you find the employees capable of working the required mathematical problems or do they need help. Check one: capable () need help ().
6. Do you feel that employees in the semi-skilled and unskilled jobs, in general, show a need for a better background in mathematics? yes _____ no _____
7. Check the level of ability in the following operations of mathematics as they apply to your employees:
 capable - (can work required problems without instructions)
 weak - (can work required problems if given some instruction)
 need help - (lacks knowledge necessary to work most problems)

	Capable	Weak	Need Help
Addition			
Subtraction			
Multiplication			
Division			
Fractions			
Percentage			
Algebraic operations			

8. Is a lack of ability in mathematics a common reason for an employee not advancing to a better job? yes _____ no _____
9. Do you feel that a unit in mathematics in a high school industrial arts course, using problems related to industrial processes, would better qualify men for the semi-skilled and unskilled positions in industry? yes _____ no _____